The Royal Institution
Science Lives Here
Masterclass network

OTS Masterclass - Rabbits and Sequences

Session Leader Notes

Inspiration:

An exploration of number sequences: some which occur naturally and others which can be constructed - and how they sometimes turn out to be the same! We can understand and make predictions about different real-world process by looking at the sequences of numbers they produce, and finding rules which govern their behaviour.

Overview of Activities:

- Discussing sequences, and finding examples
- Making predictions about number sequences
- Rabbit populations activity
- 1p and 2p coin activity
- Additional activity/if time/extension activity Prime numbers - patterns and predictions

General Masterclass resources needed:

- Register of children
- Consent forms and emergency information to hand
- Stickers and markers for name badges
- Adult register
- Ri child protection policy
- Paper and pencils/whiteboards for workings
- 2 different coloured post it note pads
- Drinks and biscuits

Specific resources needed (printable worksheets in worksheet folder):

- Calculators
- 1p and 2p coins - around 10 1ps and 5 2ps per student; coins can be obtained from the bank in bags of £1 - two bags of 1ps and two bags of 2ps will be enough for 20 students
- Squared paper
- Whiteboard or flipchart at the front of the room, with appropriate pens
- Copy each of 'find the next number in the sequence' worksheet
- Copy each of Rabbit Populations worksheet and Rabbit Populations Extension half-sheet
- Copy each of prime number grid worksheet

Support resources:

- Helper notes: An overview of the Masterclass content and activities
- Supporting notes: Extra information and background on number sequences and the specific examples used in this Masterclass
- Session script: Suggested wording for each section of the session

Things to prepare in advance

- Print worksheets and resources as detailed above
- Gather general Masterclass resources
- Practise saying 'Eratosthenes'

Ask the Ri

Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk

Feedback

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

Time plan of Masterclass:

<table>
<tr><th>Slides & Time</th><th>Overview</th><th>Activity</th></tr>
<tr><td>Slide 1</td><td>Title slide</td><td>If you wish, replace this with Slide 8 to get them started on working out some sequences.</td></tr>
<tr><td>Slides 2-7

5 minutes (5)</td><td>Introduction to the Ri

[Only include these slides for the first session in the series – otherwise remember to hide the slides before you start the Masterclass]</td><td>Use these slides to introduce the students to the work of the Ri and other ways they can get involved – see notes on the slides for more detail. In particular:
The Ri is a science communication charity which has been around since 1799. We've got a huge amount of history and lots of famous scientists lived and worked at the Ri. Most importantly, we've always been about communicating science to the general public – and that's something we still do today. We do talks and activities for the public as well as with schools all across the UK.
There are lots of family events at our building in London, including family fun days and holiday workshops just like the Masterclasses.
The CHRISTMAS LECTURES are for young people and are on television at Christmas time, looking at a different topic every year. We've got an archive on our website of all of the recent series plus many of the older ones. The CHRISTMAS LECTURES are what started the Masterclass programme. See slide notes for links.
We have a YouTube channel with lots of videos for people interested in science (and maths engineering, computer science…), especially our ExpeRimental series which is all about doing experiments at home.
Students are part of a big family of Masterclass attendees – we have been running Masterclasses since 1981.
Students at series running within reach of London will be invited to a Celebration Event at the Ri in June/July.
You can become an Ri Member to get more involved with what we do (and enter the ballot to buy tickets to the CHRISTMAS LECTURES filming).</td></tr>
<tr><td>Slide 8

10 minutes (15)</td><td>Introduction: finding the next number in a sequence, and working out the reason. Instructions on screen.

Students completing worksheet – encourage discussion! Adults circulating and chatting with students</td><td>*GIVE OUT WORKSHEET 01 "find the next number in the sequence"*

Students should complete each row with what they think comes next. It's important the students explain why they think their answer is right.

If students finish all the examples, they should be encouraged to write down their own sequences, and explain why they are a sequence in the same way.</td></tr>
<tr><td>Slides 9 - 11

10 minutes (20)</td><td>Discussion of introductory work – whole group</td><td>Discuss the answers given in the first activity. Some answers may have been given that are not the ones given in the slides - but as long as the student has a good reason for it, their answer is correct.

Animations:
The sequence appears, then the next two numbers in that sequence, then the rule. The sequences are in the same order as the worksheet, spread over three slides.

Students who have invented their own sequences can give their examples (if they haven't got any, give them a little more time now to do this) for you to write up on the board.</td></tr>
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Slides 12 – 18 5 minutes (25)	Look at term-to-term rules and how to write the rules for finding the next term in a sequence. Introduction of terminology. Whole group together – if needed, give examples to clarify your explanations on a separate board (this could take longer).	(Slide 12) Explain that finding the reason for the next term in a sequence is called a "**term-to-term rule**" and that we want to be able to write this down mathematically. The rule should give us the information for how to get to the next term in the sentence and in order to do this without a long sentences we will need to use some terminology. (Slide 13) Explain the **position number** – the number which tells you which position each term in the sequence has. We label this "n", and it can point to any position in the sequence. (Slide 14) Explain that the numbers in a sequence are called **terms**, and we use notation to be able to point to each term – if we label the sequence "a", the 1^{st} term is "a_1", etc. Explain what a subscript is and how it is used as a label. (Slides 15 & 16) Explain that to go to any unknown number in the sequence we use the label "n", and call it the "**n^{th} term**", a_n. Show the students what it means when n=1, n=2, n=3 etc. (Slides 17 & 18) Explain how we use this notation to talk about the terms before and after the nth term; a_n is the n^{th} term, so a_{n-1} is the term before, etc. The "n", "n-1" and "n+1" are labels, not sums – they tell you which term to look at. Show the examples for the sequence in the table.
Slides 19 – 22 20 minutes (45)	Practice finding the first term and term-to-term rules for each sequence. Introduce on board; students discussing and working on the problems using previous worksheet, adults circulate and help. Answer discussion as a whole group.	Tell the students to look back at their sequences worksheet and try to find some term-to-term rules using this notation. Explain what they will need for their rules to make sense (i.e. for someone else to be able to use them – slide 19). Discuss why you need to know the first term (as otherwise you will not know where to start – different first terms give different sequences). Talk through the first example with them (slide 20). Give the students time to write a_1 for each of the sequences, and to try to write the term to term rule (this is very challenging and might not be worth lingering over – depending on your students, you may want to stop after a short time and go through it together on the board). Then go through the answers together (slides 20-22 = they already know the next terms, so the animations bring up a_1 and the rule). Note that two of the sequences have different rules for odd and even terms – explain that this is OK (you may need to help them more with these ones). It is easier to write a rule for the square numbers in terms of n than a_{n-1}, but make sure they do the latter – they are looking for the term-to-term rule.

Slides 23-26 5 minutes (50)	Look at position-to-term rules, where the terms of a sequence can be worked out by knowing the position. Whole group – students discussions and answers given to the speaker, not on worksheets	So far we have thought about how to find the next number in the sequence from the one we already have, using a "term-to-term" rule. Some of these sequences can also be written using a general rule (for the nth term). Here the rule is relating the number in the sequence to its POSITION. Two examples are given on the slides. This is called the **nth term rule or the "position-to term" rule** because **n** is the position number. Slide 23 shows the sequence we were looking at in the previous example on the board. If n is the position number, can we use that to find the rule? Get the students to come up with suggestions linking n to a_n – some will spot that a_n is nx5 (animations show a_1, a_2, a_3 and then the formula for a_n). Ask them to use this to work out the 100^{th} term (slide 24). Discuss whether you still need to know the first term if you have the position-to-tem rule. Use slides 25 & 26 to look at the square numbers example – ask them to work out the position-to-term rule and give a reason for it, and then to find the 100^{th} term. Use hands up or give discussion time before asking for answers. State how for some sequences you can easily write both kinds of rule, but for others it is more difficult. *Possible extension if you have time: see it they can come up with position-to-term rules for the rest of the example sequences (this may not be achievable for all of them).*
10 mins (60)	BREAK	Drinks and biscuits and comfort break
Slides 27 - 32 15 minutes (75)	Rabbit populations activity Introduce problem and then helper and speaker circulate and chat with students Students compare answers to check they have understood the rules correctly	Explain that the mathematician Leonardo of Pisa modelled rabbit populations using the simplified model given on the slides and worksheet. Emphasise the rules that apply (slide 27). ***GIVE OUT WORKSHEET 02 "Rabbit Rules"*** Explain that you are starting with one pair of baby rabbits and using the rules to work out the rabbit populations after each month. Go through the first couple of months together (slides 28-31). Display slide 32, summary of rules. Ask them to continue to work out the number of rabbit pairs each month (adults, babies and total), starting from one pair of baby rabbits, and once they've finished compare with their neighbour - if they get a different set of numbers, they can check they've followed all the rules correctly. They need to remember that any pairs of baby rabbits that have just become adults won't produce any baby rabbits until the following month. Students should then try to work out a rule for how the number of rabbits changes, and discover that the total number of rabbit pairs each month is the sum of the previous two months.

	Extension – Lucas numbers. Students to use the same rules with different starting numbers.	***EXTENSION: GIVE OUT WORKSHEET 03 "Rabbit Rules 2"*** When students finish this activity, ask them to use the extension half-sheet to explore what happens if you start with different numbers of rabbits - **one pair of adults and two pairs of baby rabbits**, or other combinations. Do the rabbits die out? Is the rule for getting the next numbers of rabbits the same? NOTE: The number of starting rabbits is NOT written on the sheet – you will need to give this to them. If they start with 1 pair of adults, 2 pairs of babies, they will get a sequences called the **Lucas Numbers**.
Slides 33 - 40 10 minutes (85)	Fibonacci and Lucas numbers – discussion of what the students found, how it works, and the introduction of Fibonacci Sequence	Go through the answers to the initial problem, starting with one pair of baby rabbits (slide 33). Help the students to understand and justify why these rabbit rules lead to this sequence. At the end of any month the number of rabbits will be the babies (equal to the number of pairs 2 months before, who can all have babies now) plus all the pairs from 1 month before (who are still alive, but not all able to have babies yet) – see diagrams on slides 34 – 37. Discuss the sequence found in this activity – it's called the **Fibonacci numbers/Fibonacci sequence** (Leonardo of Pisa was also known as Leonardo de Fibonacci) starting from 1 pair of rabbits. This is a famous sequence which turns up in a lot of places. Go over the rules (which some of them will have worked out) and show them the picture of Fibonacci (slide 39). The Lucas numbers are a similar sequence to the Fibonacci numbers. They follow the same rule as the Fibonacci sequence but with different start numbers – instead of starting with 1 pair of baby rabbits, they start with 1 pair of adults and 2 pairs of babies. This gives a first term value of 3 (3 pairs in total) and the second term is 4 (3 adult pairs, as the babies have grown up, and 1 baby pair, as the existing adults have given birth). Use this to discuss why it is important to know the starting terms for term-to-term rules.

Slides 41 - 45 25 minutes (110)	1p and 2p coins activity Introduce problem and then helper and speaker circulate and chat with students Students compare answers to check they haven't missed any Patterns we found in this activity Discussion as a group	Moving on to a different activity now- give each student 10p in 1p coins and 10p in 2p coins (15 coins altogether). Challenge them to find all the ways to make a line of coins totalling a given amount in pence using these coins, **counting different orderings as different lines of coins** - work through the examples of 0p, 1p, 2p and 3p all together to clarify this (slide 41), and then let them continue for 4p,5p,6p Students should use squared paper and find a way to record their results systematically. Once they've found all the ways, ask them to compare with their neighbour - they may have missed some. What pattern can you see in the sequence of numbers you get, for the total number of possible ways? Some students may have skipped ahead and filled in the numbers once they saw the pattern. Why did they do this? Discuss the difference between seeing something that looks like a pattern, and understanding why a pattern is forming so we can convince ourselves and others it will hold. The ability to prove something is a special feature of mathematics, and students should ask why the sequence appears to be going that way. Give them a little time to discuss this amongst themselves, starting from the question, 'do any of the lines of coins you've made share anything in common with other lines of coins? They should find it is the Fibonacci sequence again (1 way to make 0p, 1 way to make 1p, 2, 3, 5 and so on). Answers on slide 42. Why does this work? See if the students can explain that (e.g.) the number of ways to make 5p is the same as the ways to make 4p with a 1p added, and the ways to make 3p with a 2p added. This will always give us all the possible ways to make that number, as these are the only two ways we can change the coin sequence by adding a coin. Use the explanations on slide 44 to help discuss this, and relate it back to the term-to-term rule for the sequence (slide 45).
Slides 46-50 10 minutes (120)	Feedback, tidy up, questions time Ask the Ri Possible NRICH problems related to Fibonacci numbers and sequences – use as extension activities or for them to do at home	Recap of session contents. Don't forget to collect any questions and feedback on post-it notes, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk nrich.maths.org/2338 Go Forth and Generalise nrich.maths.org/11164 Fibonacci Surprises

Extra activity Slides 51 – 52 10 minutes	Extra activity: Prime numbers activity - sieve of Eratosthenes	Discuss prime numbers with your students. How is a prime number defined? Can we make a list of prime numbers? ***GIVE OUT WORKSHEET 04 "Prime Number Sieve"*** Explain the Sieve of Eratosthenes is a way to find them all, by eliminating factors: First cross out all the numbers (apart from 2) that are divisible by 2; then all the numbers (apart from 3) that are divisible by 3, and so on. The numbers divisible by 4 will already be crossed out - why? (They can do these first few steps with you) Give students time to cross out everything that's not prime in the grid of numbers 1-100 on the worksheet. Can they see a pattern in the numbers? Try circling the same numbers in the other grids, to see if there are any patterns.
Slides 53 - 55 5-10 minutes	Prime numbers discussion Discussion as a group, with answers from hands up	Show them the completed grids, and discuss any patterns they have found: In general, we don't have a mathematical rule to predict the next prime number. Some patterns exist - in the grid of width 6, all the primes lie in columns 1 or 5. Can we see why? What do all the numbers in columns 2, 3, 4 and 6 have in common? A way to predict the prime numbers would be immensely powerful - prime numbers are very important in maths, and can be used to encrypt data online and underlie lots of important mathematical concepts. There's even a prize if you can work it out - finding a pattern in prime numbers is one of the Millennium Prize problems for which a prize of $1m is offered (more information is given in the additional info sheet).