Rabbits and Sequences Masterclass

Thanks for helping with this Masterclass session! Your support is much appreciated.

The session leader should be able to tell you more about the content of the session, and exactly how they'd like you to help, but this sheet should give you some basic information you may find useful. If any of this seems obvious to you, that's great!

In general, for Masterclass sessions:

- While the session leader is talking to the group, don't interrupt them or distract the students unless something is wrong that needs fixing urgently. You should also watch and pay attention to what they're saying, to set a good example.
- If things need handing out to the students, wait for the session leader to signal you to do this, as it can distract the students if you start to hand things out before they're ready.
- If the students are given a task to work on, you should circulate the room to talk to the students. Wait until they've had a chance to tackle the problem before you interrupt them, and encourage anyone who looks like they haven't started yet.
- Try not to give away the answers to the students, especially if they're working on the problem and about to discover it for themselves - if they are really struggling, you can give them a hint or suggest where they might start looking.

<u>In this session:</u>

This workshop is an exploration of number sequences: some which occur naturally and others which can be constructed - and how they sometimes turn out to be the same. We can understand and make predictions about different real-world process by looking at the sequences of numbers they produce, and finding rules which govern their behaviour.

The main activities are:

1. **Finding the next term in a sequence of numbers**
2. **Working out ways to describe the rules - both for how to get from one term to the next, and how to find a given term without knowing the one before**
3. **Describing how rabbit populations grow using a simple model**
4. **Finding patterns in the number of ways to arrange 1p and 2p coins**
5. **(If time) Making a list of prime numbers and looking for patterns in them**

There is a great deal more background information available on a separate sheet, if you would like more detail. Please ask the session leader if you'd like to see it.

Thanks again for your help with this session! If you have any other questions, please ask the session leader.

Please turn over for more detailed notes.

'Find the next number in the sequence' activity

Students will be given a worksheet with examples of number sequences. This may be handed out before the session introduction, as a starter activity. The challenge is to find the next two terms in each sequence, and to explain why. If students fill in the numbers but don't write anything underneath, encourage them to think about why their answer is correct and what the rule is for that sequence.

Some students may find answers that are different to the ones given in the slides, but as long as they have a reasonable rule for why the sequence works, their answer is still correct.

After the session introduction, the session leader will gather students together to discuss the answers, and to try to find a more mathematical way to write the rules for this sequence. Initially, discussion will be focused on finding a rule to give the value of the current term in a sequence from the value of the previous term. These are known as "term-to-term" rules.

The notation a_n for the n^{th} term of the sequence will be used, so encourage the students to use this notation. As a reminder, the 'position number', "n", is where the term is in the sequence – i.e. the first number in the sequence is the 1^{st} term, and has position number 1; the notation for that term is a_1.

Ensure the students are focusing on the term-to-term rules at this stage (some sequences might have an easier rule using another method, which we will come onto later). Some sequences have different rules depending on whether the position number "n" is odd or even. Students may also be asked to come up with their own sequences and find the mathematical rule to describe them.

Solutions will be given on the slides, but are here for your reference – the important thing for the students is to check their answers (i.e. put in the numbers for terms further along in the sequence to see if it still works).

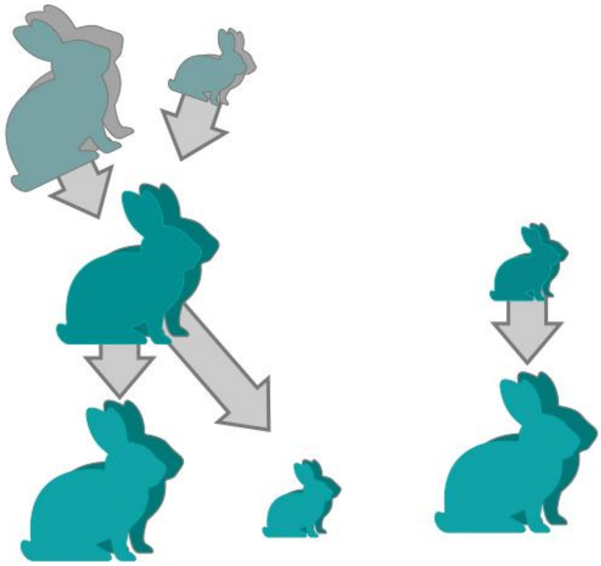
5, 10, 15, 20, 25, 30, … Rule: $a_1 = 5$; $a_n = a_{n-1} + 5$
1, 3, 5, 7, 9, 11, 13, … Rule: $a_1 = 1$; $a_n = a_{n-1} + 2$
2, 4, 8, 16, 32, 64, … Rule: $a_1 = 2$; $a_n = 2xa_{n-1} = 2a_{n-1}$
1, 4, 8, 11, 15, 18, 22, 25, … Rule: $a_1 = 1$; when n is even: $a_n = a_{n-1} + 3$, when n is odd: $a_n = a_{n-1} + 4$
1, 3, 6, 10, 15, 21, 28, … Rule: $a_1 = 1$; $a_n = a_{n-1} + n$
4, 5, 3, 6, 2, 7, 1, 8, … Rule: $a_1 = 4$; when n is even: $a_n=a_{n-1} +(n-1)$, when n is odd: $a_n=a_{n-1} +(n+1)$
1, 4, 9, 16, 25, 36, … Rule: $a_1 = 1$; $a_n = a_{n-1} + 2 \times n - 1 = a_{n-1} + 2n - 1$

We will now think about a general formula for the n^{th} term, so that you can find the term value without knowing the term before - for example, if we wanted to know the 100^{th} term without working out all 99 terms before that. We will go through a few examples on the board, and if there is time the students may look at some of the sequences they have been working on to see if they can find these rules. This is difficult for some of these sequences. See the background information sheet for the answers.

Leonardo's Rabbits

The next activity focuses on a way of modelling rabbit populations, developed by Leonardo of Pisa (also called Leonardo de Fibonacci - although you may want to keep this a surprise for any students who have already seen Fibonacci numbers).

The rules for the next generation of rabbits can be difficult to follow, so make sure you familiarise yourself with how it works beforehand. One pair of baby rabbits exists initially. Each month, any baby rabbits mature into adults, and a pair of baby rabbits is born to each pair of adult rabbits - but not to ones which have just matured. It's easier to consider the number of baby pairs that will be born first, and then to note that the baby rabbits mature after that, to avoid any confusion.



We've asked the students to compare their answers with a neighbour once they've completed the table, as if they've misunderstood the rules, or made any arithmetic errors, this can lead to further errors later so it's important this is noticed before they try to study the resulting sequence.

The rabbits are always considered in pairs, so the numbers they're looking for are the number of pairs of rabbits, not the number of rabbits - make sure this is clear, and if anyone writes down numbers twice as big, remind them of the column headings which are phrased in terms of 'pairs of rabbits'. They're also looking for the total number of pairs in each generation - adults and babies.

Students may notice that the total number of rabbit pairs gives the Fibonacci sequence, and if they have seen it before, warn them not to give anything away to others around them. Their task is then to write a rule for how the number of rabbits changes each month.

If any students finish this task early, they can try it starting with other numbers of rabbits - starting with one pair of adult rabbits and two pairs of baby rabbits produces the Lucas numbers, and other combinations students choose will give sequences with the same rule about how the total number of rabbits changes.

Coin Combinations

The next activity involves using a pile of 1p and 2p coins to make lines of coins which add up to a given total. The first few examples (total of 0p, total of 1p, total of 2p and total of 3p) will be discussed together, then the students can carry on to work out further totals.

The aim is to find all the possible ways to line up any number of coins to make the total, and the same coins arranged in a different order count as separate answers. We're asking the students to use squared paper to record their results systematically, so you may find a variety of ways to present the answers, which may not seem to you the best way, or match what you would do - but what matters is that the student can accurately record, understand and interpret their own findings.

Some students may quickly find that the sequence is the same as the Fibonacci sequence. This might mean they jump ahead and fill in future answers without knowing for sure that this pattern will continue. Discussion on this will follow the activity, so you don't need to go too deeply into it with the students individually, but if you see a total number of ways written without the different ways drawn out as a list, you should challenge them to complete the list - it's good that they can see a pattern, but that just gives them a target of how many to find, and they need to back up their prediction with an answer.

We're also encouraging them to discuss their answers with a neighbour, as it's easy to miss one of the possibilities when listing all the combinations, so hopefully any they've missed will have been found by their neighbour and this will help them to get a complete list. They can also try to find a systematic way to list all the possibilities - finding all the sets of coins that work, then working out how many different ways they can be put in order.

We've suggested giving each student 10 1p coins and 10 2p coins, which should be enough to continue working on this problem as long as it'll be running - they might work entirely without the coins, only writing answers down, but there won't be enough coins to make all the combinations at once, beyond the first few sets. By the time they reach larger sets they will hopefully have developed an approach that allows them to generalise without using the physical coins, but if not they can borrow some more if someone nearby doesn't need theirs.

Prime Number Grids

Extension activity

This activity may be included as an extension if there is time. It's all about trying to predict patterns in numbers which don't have a known pattern - prime numbers. We'll be using a 10 by 10 grid of numbers to first of all identify all the primes, by crossing out all the multiples of 2 (except 2 itself), then all the remaining multiples of 3 (except 3 itself), and so on.

Students will find all the multiples of 4, and some of the multiples of 5, and all the multiples of 6, will already have been crossed out by the time they reach them. They won't need to go beyond multiples of 7 to fill the whole grid, but some may want to check - but don't let them waste too much time on this. Once they've crossed out everything except the prime numbers, you can tell them they've finished.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
91	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

The next activity is to circle only the prime numbers in the lower grids - keep an eye out for silly mistakes! There should be an obvious pattern in the lower right grid, which is 6 squares wide. The prime numbers should only be found in columns 1 & 5. This happens because anything outside of columns 1 and 5 will be divisible by 2, or 3. If students see this pattern, ask them to think about why so that they can feed back in the discussion.

The grid of width 7 won't have the same obvious pattern in the columns, but a similar pattern occurs diagonally in the grid, and it can be explained by reference to the grid of width 6 - each row is one square wider, so the lines get spread across as a diagonal, as does the gap.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49
50	51	52	53	54	55	56
57	58	59	60	61	62	63
64	65	66	67	68	69	70

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60