Möbius Bands Masterclass: Session Script

The Royal Institution Science Lives Here
Masterclass network

This icon means there's a slide, or slides in the presentation to accompany this line of the script.

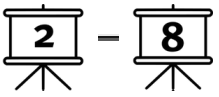
This icon indicates the students will have an activity to do, or something to write.

Introduction (10 minutes)

Welcome to today's Masterclass. To start with, there's an activity for you to work on - cut your sheet of A4 paper into long strips 3cm wide. Make sure they're the same width all the way along! How many strips can you make? Where do you need to mark the page to be sure your strips are 3cm wide all the way along?

[This next section can be skipped if this masterclass is not the first one in the series.]

These masterclasses are organised by the Royal Institution. Has anyone heard of the Royal Institution before? It was founded in 1799, and has always been about letting everyone have access to science - organising masterclasses and lectures, including the Christmas lectures.



Many famous discoveries have been made in the Faraday building where they are based, including 10 chemical elements. Michael Faraday, who the building is named after, did work there on electricity and optics, which we all use every day.

The Ri are perhaps most famous for Christmas Lectures for young people, which have taken place since 1825. They have been televised for several decades, and many past series are available on the Ri website. The Masterclass programme was born out of the Christmas lectures delivered by Christopher Zeeman in 1978, which were the very first ones on mathematics!

There are many opportunities for you to visit the Ri building in London, and see

the historic rooms for yourself. There is a small museum too. There are lots of talks and holiday events for young people: all the details are on the Ri's website.

Mobius Bands Investigation (15 minutes)

You've been asked to cut your sheet of paper into long strips that are 3cm wide.

- **How many strips did you manage to cut using this one piece of paper?** You should be able to make 7 strips, because the paper is 21cm wide.

Let's examine one of these pieces of paper we've just made.

- **First of all, how many faces does the piece of paper have?** It's not a trick question - like all pieces of paper, it has two faces. **How could we tell?** Mark 1 on one side and 2 on the other?
- **Secondly, how many edges does it have?** It's a rectangle, so it has four edges. **What about just the long edges?** It has two long edges, one down each side.

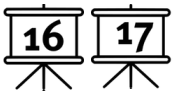
We're going to use this piece of paper to make a **Möbius Band**. It's a shape that's named after the mathematician that discovered it, August Mobius.

To make one, bring the ends of your strip of paper around as though you're going to make a loop - a loop like this where the ends join up is a shape you may already have a name for - **can anyone tell me what it's called?** It's a cylinder, although it may not look like one as you're used to cylinders being long and thin, while this one is very short.

In order to turn it into a Möbius band, you need to take one side of the paper and turn it over. Then you should stick it down and make sure you use plenty of glue/tape on both sides to make sure it doesn't come undone.

In order to investigate this shape, we're going to start by doing two things. I'll tell you what the things are first, and then you should do them and see if you can answer the questions underneath.

First, you should draw a line **down the middle** of the Möbius Band, without taking your pen off the paper - or if you do, starting again from where you left off. If you have tape on your Möbius Band which stops the pen showing up, just ignore it - keep going



through, and keep going until you get back to where you started. Then see if you can answer this question: how many faces?

Then, draw a line **down the edge** of the Möbius Band, again without taking your pen off the paper, and keep going until you get back to where you started. Then see if you can answer this question: how many edges?

Investigate both of these things and we'll come together afterwards to discuss our answers.

Mobius Bands Discussion (5 minutes)

I hope you have answers to the questions on this slide. Does anyone think they can answer the first question - **how many faces?**

It looks like the Möbius Band only has one face - how do you know it only has one face? Can you be sure? You can draw a line all the way along it without taking your pen off, and cover the whole surface of the Möbius Band. So it must only have one face. Why does this happen?

We started with a strip of paper that has two faces - but we put in a twist. So one face of the paper has joined onto the other, and the other face has joined back onto the first. So this shape only has one face - the paper now only has one side.

- **If we were to use a coloured pencil to colour in the Möbius Band, how many times would we need to lift the pen to colour in the whole shape?** We wouldn't! It can all be done in one go.

What about this next question - **how many edges?**

If you draw a line along the edge, and don't take your pen off, you can draw all the way along the edge of the shape and there's no edge not covered when you get back to where you started - this means it's all part of the same edge. So it only has one edge!

We started with a strip of paper with two different edges, but our twist has joined one edge on to the other edge, and the other edge back on to the first edge - so they're now all part of the same edge.

So this shape is very interesting - it's got one face, and one edge. Möbius thought it was very interesting and worth studying, and it's part of a branch of maths called Topology which looks at interesting shapes and what makes them interesting.

Next I'd like you to draw a little ant on your Möbius Band.

- **If your ant were to walk around the Möbius Band, all the way round and back to where it started, how far would it have walked? Think carefully about your answer.**

Your strips of paper are around 30cm long, so if it walked for 30cm it would be back in the same place, but on the other side of the paper - so it would need to walk around 60cm to get back to this position.

More Twists Investigation (15 minutes)

We made Möbius Band with a half-twist, by turning over the piece of paper once. But this is not the only kind of Möbius Band we could make! You should still have 6 pieces of paper left from your original sheet - take four of them now and here are the things I want you to make. Wait until I've told you all of the things to make first, and then I'll say go and you can start.

First, a no-twist Möbius Band. This should be easy enough.

Then, I'd like you to make a Möbius Band with not a half-twist, but a full twist - so you should start with the ends of your paper together like this, and if you turn it over once that's a half-twist, and you'll need to turn it over again to get a full twist. Don't let go until you've stuck it down!

I'd also like you to make a Möbius Band with 1½ twists - how many times would we need to turn it over to do this? Three times. And a Möbius Band with two full twists, which means you need to count 1, 2, 3, 4 turns over.

Use four strips of paper to make all of the things in black. **Go!**

Now that you've made all of these shapes, we're going to use these worksheets to write down all the things we can find out about them. For each one, how many faces it has, and how many edges.

Complete the first five rows of the table - you already know the answers for the second row!

More Twists Discussion (10 minutes)

Have you completed the table? It should look like this.

There's an interesting pattern here! **Can anyone describe anything about the patterns in the numbers?**

- The number of edges is always the same as the number of faces
- It alternates between 1s and 2s

- You can't ever have more than 2 faces, or 2 edges
- If there's a whole number of twists, or an even number of half-twists, it has two faces/edges, and if there's an odd number of half-twists it has one.

Can we explain why this happens? When we made one half-twist, we took the two edges and two faces and joined them on to each other, so the original Möbius Band has only one of each. But if we do a full twist, we join each face and each edge back onto itself, so there are still two faces and two edges!

As you put in more twists, the pattern carries on - one face/edge if the edges are joined on to the opposite ones, and two if they're joined back onto themselves, although there will be more full twists in between. The faces will be twisted around each other, but you can only colour in one side of the paper without taking your pen off.

Cutting Mobius Bands Activity (15 minutes)

The next thing we're going to do involves your pair of scissors again. I'd like you to watch carefully. Using your original Möbius Band (the one you first drew an ant on), pinch it together at one end and use your scissors to make a single cut on the line you drew down the middle of the band. If you open this up you now have a hole to start cutting from.

Cut along the line you drew, all the way around the band. If you reach the tape, just go straight through. What happens?

You might find this behaves in a slightly unexpected way - if you cut a Möbius Band in half, you won't get two pieces! You will have one long piece that's twisted.

Using the bottom three boxes on your worksheet, write in the first box "½, cut in two". Work out how many faces and edges this shape has, and write them in the boxes too.

This shape should have two edges and two faces. So it's not an ordinary half-twist Möbius Band!

We're going to try this again, but cutting the shape up differently. You'll need to make another two Möbius Bands, using your two remaining strips of paper - but before you make them, we're going to mark on some different lines to cut along.

Take one of your strips and divide it into three equal parts - **how many centimetres wide will each one be?** You'll need to make some marks at the top and bottom of the paper and join

them up with a ruler. Draw the lines on both sides, the front and back of the paper.

Now take another strip - we'll be dividing this one into four, and the easiest way to do this is by folding it! Fold the strip in half down the middle, and then fold that in half again. When you unfold it, you'll see the three creases, and you should draw them in using your ruler so you have three lines on each side of the paper.

Now, you just need to make these up into ordinary, half-twist Möbius Bands.

Make the two bands, and cut them along the lines - remember to start by pinching one end and cutting in to it, and see what happens.

You might find that these are unexpectedly not one thing! So you'll need to modify the table. Use a ruler to split each of these boxes in half, so you have somewhere to write about each of the pieces.
Then you can fill in the information.

The things we make here are pairs of Möbius Bands linked together - if you cut it in thirds, you leave the half-twist Möbius band in the middle, and that's the part that still only has one face and edge - the other part is the outside of the original Möbius Band, and it's the same thing you'd get if you just cut it in half.

Cutting into quarters is the same as cutting in half, then cutting the resulting shape in half - so it'll give two shapes that are two-sided.

Square and Hearts Cutting activity (15 minutes)

To finish, we're going to do one more thing with Möbius Bands. I'm going to give everyone four pieces of coloured paper, in two different colours.

Split them into two pairs, where each pair contains two different coloured strips. With one pair, you're going to make two no-twist Bands, and with the other, make two normal half-twist Möbius Bands. **But this is important!** Your two Möbius Bands need to be twisted in opposite directions. When you make a Möbius Band you pick a direction to twist in - over towards you, or away. So you need to remember which way you twist the first one and twist the other one the opposite way.

Next, I want you to use tape or glue to attach the two no-twist bands together - back to back, and at right angles as shown in the picture. Put plenty of tape/glue on so they're secure!

Do the same thing with your two Möbius Bands - put them back to back at right angles and stick them down securely.

Before we go any further, I'm going to give you some time to discuss this between yourselves and try to work out what's going to happen - if we cut these strips in half, like you've been doing with the single Möbius Band, and cut both of them, what do you think we'll get? **Don't do it yet!** Try to imagine - think about the untwisted bands first, and see if you can picture what will happen.

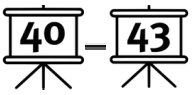
Now, using our safe starting technique of pinching the end of the strip, cut your no-twist bands in half. What do you get?

Even though a square is a shape you probably know pretty well (I hope!), it's amazing that this will give you a square. **Can you see why it's got four right-angle corners?** It's from where we cut two lines that crossed each other at right angles. **Can you try to put the original two loops back together?**

Now try the same with your Möbius Bands.

You should find you have a lovely pair of **linked hearts**! If you don't it may be because your bands weren't twisted opposite ways. **Can you see the twists?** The hearts are each made from half of one of your original Möbius Bands. **Can you find the four right angle corners again, just like on the square?**

End of session - recap



In this session we've made Möbius Bands just like August Möbius did, and investigated why mathematicians find them interesting. We've tried putting in more twists, and cutting them up in different ways, to see if that changes the way they work. And we've seen some other interesting shapes you can make.

Next time you're at home with your family and friends and want something interesting to do, get some paper and scissors and show them how to make Möbius Bands - and make sure you tell them about all the interesting mathematical properties they have!