OTS Masterclass - Möbius Bands

Session Leader Notes

Inspiration:

The Möbius Band is an example of what is known as a non-orientable surface. It has just one side and one edge. In these activities, students will investigate how Möbius Bands can be created, and explore some of the interesting things that happen when they are dissected.

Overview of Activities:

- Mobius band investigation
- More twists investigation
- Cutting Möbius bands investigation
- Square and Hearts activity

General Masterclass resources needed:

- Register of children
- Consent forms and emergency information to hand
- Stickers and markers for name badges
- Adult register
- Ri child protection policy
- Paper and pencils/whiteboards for workings
- 2 different coloured post it note pads
- Drinks and biscuits

Specific resources needed (printable worksheets in worksheet folder):

- A4 blank or brightly coloured paper (one sheet per student, with some spares in case)
- Pre-cut strips of paper in two colours
- Scissors, glue/tape, pencils and rulers for each student - glue/tape may be shared, but for everything else one each will be needed
- Möbius properties worksheet (one per student)

Support resources:

- Helper notes: An overview of the Masterclass content and activities
- Supporting notes: Extra information and background on the content of this Masterclass
- Session script: Suggested wording for each section of the session

Things to prepare in advance

- Print worksheets and gather resources as detailed above
- Practise the square and hearts demonstrations from the final section
- Gather general Masterclass resources

Ask the Ri

Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk

Feedback

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

Time plan of Masterclass:

Slides & Time	Overview	Activity
Slide 1 5 minutes (5)	Introduction Instructions on screen. Helper and Speaker circulating and chatting with students	Settling activity - measuring and cutting paper into strips Ask the students to cut the A4 paper into strips that are as long as the long edge of the paper, and 3cm wide. How wide is the piece of paper? How many strips will they be able to cut? How many places do the need to mark down the page in order to join them with straight lines? They should keep all 7 pieces of paper, as we will use them all.
Slides 2-8 5 minutes (10)	Introduction to the Ri *[Only include these slides for the first session in the series – otherwise remember to hide the slides before you start the Masterclass]*	Use these slides to introduce the students to the work of the Ri and other ways they can get involved – see notes on the slides for more detail. In particular: • The Ri is a science communication charity which has been around since 1799. We've got a huge amount of history and lots of famous scientists lived and worked at the Ri. Most importantly, we've always been about communicating science to the general public – and that's something we still do today. We do talks and activities for the public as well as with schools all across the UK. • There are lots of family events at our building in London, including family fun days and holiday workshops just like the Masterclasses. • The CHRISTMAS LECTURES are for young people and are on television at Christmas time, looking at a different topic every year. We've got an archive on our website of all of the recent series plus many of the older ones. The CHRISTMAS LECTURES are what started the Masterclass programme. See slide notes for links. • We have a YouTube channel with lots of videos for people interested in science (and maths engineering, computer science…), especially our ExpeRimental series which is all about doing experiments at home. • Students are part of a big family of Masterclass attendees – we have been running Masterclasses since 1981. • Students at series running within reach of London will be invited to a Celebration Event at the Ri in June/July. • You can become an Ri Member to get more involved with what we do (and enter the ballot to buy tickets to the CHRISTMAS LECTURES filming).
Slides 9-19 15 minutes (25)	Mobius band investigation Helper and Speaker circulating and chatting with students	First ask the students how many strips they managed to make from their piece of A4 paper. It should measure 21cm wide, so they should be able to make 7×3cm strips from each sheet. Before we go any further, ask the students some questions: how many faces, or sides, does the strip of paper have? Two. And how many edges? Some may say four, as it's a rectangle - how many long edges? Two. Ask the students to use one of these strips to make a Möbius band - show the image of August Möbius and explain that Möbius was a mathematician who discovered this shape. To make the band, bring the ends together as though you're going to make a cylinder/bracelet shape, and then turn one side over so the ends meet again but the strip is twisted. Instruct the students to glue it well, and if using tape, to put tape on both sides of the join to make sure it doesn't come undone. To investigate this shape, the students have certain tasks - the first is to draw a line down the middle of the strip - and to keep going until they get back to where they started - and then

		answer the question: how many faces does the strip of paper have? The next task is to draw a line close to one edge of the band, and again keep going until they get back to where they started. How many edges does the band have? You can put up the slide showing the tasks and questions and let the students do both of these activities in their own time.
Slides 19-21 5 minutes (30)	Group discussion: what are the properties of this shape? Speaker leading discussion	Call the group together for a discussion of what they have found - two faces, two edges has become somehow one face and one edge. How do the students know this has only one face? Can they explain why they are sure? The paper you started with has two faces, and two edges, but adding a twist has joined each face onto the other, and each edge on to the other as well. You can ask the follow-up question - how many times would you need to lift your pen if you wanted to colour in the whole Möbius band, on both sides? You won't need to at all - it can be done in one go. Ask the students to draw a little ant somewhere on the centre line of their Möbius band. If this ant was to walk along the line around the Möbius band, all the way round and back to where it started, how far would it walk? The strip of paper is around 30cm long, so if the ant walks all the way around it will walk 60cm.
Slides 22-27 15 minutes (45)	More twists investigation Helper and Speaker circulating and chatting with students	The next investigation is to make other kinds of Möbius bands and see how this changes the number of sides. Using four of their remaining 6 paper strips, the students should make a no-twist Möbius Band (also known as a cylinder!), a Möbius band with one full twist, one with 1½ twists, and one with 2 full twists. This can be thought of as 0, 2, 3, and 4 half-twists - starting from an untwisted pair of ends, turning one side over once will give the Möbius band we already have, and then additional turns will give the others. Ask them to make one of each, and label them all with how many twists - 0, ½ (for their existing band), 1, 1½ and 2. Ask the students to investigate, by drawing lines down the middle and along the edge, how many faces and how many edges each shape has. The bottom three rows of the table are for another activity later, so leave them blank.
Slide 28 10 minutes (55)	Group discussion: why do the twists cause these properties? Speaker leading discussion	Bring the students together to discuss their findings. How many edges and faces does each type of Möbius band have? Are there any patterns in the table? Why does this happen? As discussed earlier, the original unattached strip of paper has two faces, and two edges. When we put in a half-twist, we join the front of the paper on to the back, and the back on to the front - so they become one face. Similarly, we join the left edge on to the right and the right edge on to the left, so they become one edge. If we put in another half-twist, this will swap the edges and faces back again - so instead of having only one, this will go back to having two. The faces will be twisted around each other, but you can only colour in one side of the paper without taking your pen off.

		The pattern continues - where you have an odd number of half-twists, there's only one face and edge, and when the number of half-twists is even, you have two of each.
10 mins (65)	BREAK	Drinks and biscuits and comfort break
Slides 29-35 15 minutes (80)	Cutting mobius bands activity Helper and Speaker circulating and chatting with students	Next we're going to use the bottom section of the worksheet. Ask the students to take their half-twist original Möbius Band, which should have a line down the middle of it. Carefully, they should pinch the band back on itself and make a small cut into one end, on the line. Then they can open this and use the gap they've just made to cut all the way along the middle line until they have cut the Möbius band in half. Something unexpected happens! The Möbius band does not come apart into two pieces - as you would expect it to do. It produces one single, long thin strip that is also twisted. Ask the students to write "half-twist, cut in two" on their sheet in the first empty box, and then in the same row determine how many edges and faces it has. This new object is not the same as a Möbius band, as it has two faces and two edges - one edge is the edge you have just cut along, and the other is the straight outside edge of the original band. Ask the students to take their remaining two strips of paper, and a pencil and ruler, and to divide one of them into three lengthways, drawing lines along the whole strip on both sides, and the other into four (they might find it easiest to fold it lengthways and then fold again to make crease lines they can draw along, rather than trying to measure ¾cm). Once they've done this, they should use the marked up strips to make two more half-twist Möbius Bands - and then cut them along these lines. They can use the remaining two rows for "half-twist, cut into three" and "half-twist, cut into four" to record their results. For the three and four strips, they will find there are two pieces, so they should determine for each how many edges and faces it has, and divide the box in two to mark them both. To explain these results, consider that cutting the Band in thirds is just the same as cutting the sides off the original Band, so the middle section is still just a piece of the original Band, and the other piece is what was the outside, and will be the same thing you get if you cut one in half. Cutting into quarters is the same as cutting in half, then cutting the resulting shape in half - so it'll give two shapes that are two-sided.
Slides 36-39 15 minutes (95)	Square and hearts cutting activity Activity completed all together - helper and speaker circulating and advising students	Give the students four strips of paper, two each in two different colours. Ask them to make one strip of each colour into a no-twist Möbius band (a cylinder), and the other two into half-twist Möbius bands - but making sure they make them with twists in different directions, so they need to remember which direction they twist the first one, and twist the other the opposite way. Once they have done this, ask them to place the two different coloured cylinders on top of each other at right-angles, making a double loop - and to put tape on all four joins between the two pieces (or to glue it securely in the square where they overlap). You can demonstrate this at the front. Then ask them to do the same thing with the two Möbius bands.

		Now, give the students a minute to think about what would happen if they cut the two cylinders in half - just like they did with the Möbius band earlier, by pinching it carefully and making a small cut to start, then cutting all the way round one loop, and then cutting the other strip (which will not be a loop any more) in half as well. Once they've thought, let them try it and see what happens. They get a square! See if they can piece the double loop back together. Finally, ask the students to repeat the same process with the double Möbius band. It should (if they've correctly twisted them opposite ways) give a pair of linked hearts. Can they see where each of the pieces has come from? Can they find four right-angle corners, just like on the square?
Slides 40-43 5 minutes (100)	Feedback, tidy up, questions time Ask the Ri	Recap of session contents. Don't forget to collect any questions and feedback on post-it notes, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk
Slides 44-45	Possible NRICH problems and other content related to this session – use as extension activities or for them to do at home	nrich.maths.org/7530 Cylinder cutting Videos by Vi Hart: Wind & Mr Ug bit.ly/wind-mrug Candy Buttons bit.ly/candy-buttons Möbius Music Box bit.ly/mobius-musicbox