Möbius Bands Masterclass

Thanks for helping with this Masterclass session! Your support is much appreciated.

The session leader should be able to tell you more about the content of the session, and exactly how they'd like you to help, but this sheet should give you some basic information you may find useful. If any of this seems obvious to you, that's great!

In general, for Masterclass sessions:

- While the session leader is talking to the group, don't interrupt them or distract the students unless something is wrong that needs fixing urgently. You should also watch and pay attention to what they're saying, to set a good example.
- If things need handing out to the students, wait for the session leader to signal you to do this, as it can distract the students if you start to hand things out before they're ready.
- If the students are given a task to work on, you should circulate the room to talk to the students. Wait until they've had a chance to tackle the problem before you interrupt them, and encourage anyone who looks like they haven't started yet.
- Try not to give away the answers to the students, especially if they're working on the problem and about to discover it for themselves - if they are really struggling, you can give them a hint or suggest where they might start looking.

In this session:

This workshop is an exploration of the Möbius Band, an intriguing shape made simply from a piece of paper with a twist. The Möbius Band is an example of what is known as a non-orientable surface, and it has just one side and one edge. In these activities, students will investigate how Möbius Bands can be created, and explore some of the interesting things that happen when they are dissected.

The main activities are:

1. **Mobius band investigation**
2. **More twists investigation**
3. **Cutting Möbius bands investigation**
4. **Square and Hearts activity**

There is a great deal more background information available on a separate sheet, if you would like more detail. Please ask the session leader if you'd like to see it.

Thanks again for your help with this session! If you have any other questions, please ask the session leader.

In this session:

Introductory activity: cutting strips of paper

The students will be asked to cut a piece of ordinary A4 paper into long strips 3cm wide. They will be given rulers and pencils, and the aim of this activity is to give them a simple task to focus on. They should be able to cut 7 strips from the paper, which measures 21cm on the short side. You should make sure they're not cutting the wrong way (the strips should run the length of the long side of the paper) and that their strips are measured reasonably accurately - if the first few strips are too wide, they won't have space for all 7 and we will be using all of them in the session.

The session leader should have brought some spare blank strips in case of this happening, so during the session if you see the students using more strips than is specified for an activity, or not having enough, you can fetch them these extra strips to save them cutting more.

Möbius Band Investigation

Students will be asked to examine the strips of paper they've created - how many sides do they have, and how many edges? Some may answer that there are four edges as it's a rectangle, but if asked how many long edges they have, there are two.

In all the discussion of faces and edges on the strips, we are carefully sticking to the word 'face' to describe a flat side of the paper, and 'edge' to describe the cut edge - the word 'side' can reasonably be used to describe both, so its use should be avoided if possible to prevent confusion. Also, 'faces' and 'edges' are standard terminology when describing 3D shapes and it will be useful to cement this wording.

The students will now use one of their strips to make a Möbius Band. They should be shown how to do this by bringing the ends of the strip round as though to make an untwisted loop, then one side should be turned over exactly once. This will produce a half-twist in the strip, and that's the thing we're looking for at this stage - in later activities more twists will be introduced.

Students should be encouraged to use plenty of tape/glue to stick down this join - later, when we're going to cut things up, bands can fall apart if not sufficiently glued, so without revealing anything about why, make sure you encourage them to stick it down well - with a piece of tape that wraps round and covers both joins, or a separate one on both sides.

The students will have two questions posed and then be allowed to investigate for themselves, given the directions to draw a line down the middle of the strip, and then to determine how many faces it has, and to draw a line along the edge of the strips, and to determine how many edges it has. If they've used tape to stick their strips, they won't be able to draw on the part of the strip with tape on. If they query this, tell them to draw the line across the tape and pretend it's not there.

Drawing these lines should guide them towards the answers, but you can ask them questions to help - how many times did you need to take the pen off the paper to draw the line (clarify the difference between times when you needed to, and times when you took it off to make drawing easier, and put it back in the same place)? They should be able to do the whole thing without taking the pen off. How much of the strip was not drawn along by your line? What does this mean about the number of faces/edges?

The session leader will now bring the students together to discuss their findings, and ask them to draw an ant on the paper strip, so they can imagine it walking around. The ant doesn't need to be very big, or detailed, and students concerned about their ability to draw a realistic ant can be reassured that a dot, or a dot with legs is fine instead (if it comes up, ants have six legs as they are an insect).

They will also be asked how far the ant would have to walk if it walked all the way along the line - if students ask, the strip of paper is around 30cm long. It is actually 297mm if anyone wants to give a more precise answer, but for our purposes 30cm is sufficient.

More Twists Investigation

Next the students will use four more strips to make Möbius Bands with other numbers of twists (strictly the term Möbius Band refers to a strip with a half-twist, but the term is being used for all of the similar objects, including the 'no-twist Möbius Band', which is technically a cylinder.)

It can be unclear when referring to 'the number of twists', as each time you turn over the paper you add a half-twist, and two turns over of the paper will give one full twist - so the 'number of twists' is half the 'number of times you have turned over the paper'. The students will need to make Bands with 0, ½ (their existing band), 1, 1½ and 2 twists, and to investigate in a similar way how many faces and edges they have.

The bottom three rows of the table on the worksheet are for another activity, and should be left blank for now.

They will be asked to discuss their results and seek patterns in the table, and there are a number of valid patterns to note.

- The number of edges is always the same as the number of faces
- It alternates between 1s and 2s
- You can't ever have more than 2 faces, or 2 edges
- If there's a whole number of twists, or an even number of half-twists, it has two faces/edges, and if there's an odd number of half-twists it has one.

Other patterns may be described also, but these will help them to understand why this happens - a full twist or a multiple of a full twist restores the connection of faces and edges to the sides they were originally on, so it gives an object with the same number of faces and edges as the original strip of paper. An extra half-twist will connect these to their opposites, so one face/edge results.

The bands with one and two full twists will still have two distinct sides that can be marked separately, although they will be twisted around each other and especially in the two-twist case the object may be difficult to manipulate. Some students may accidentally tear their bands, in which case you could provide a new one and make it up for them if they wish, to save time.

Twists before sticking	Number of faces	Number of edges
No twist	2	2
½ twist	1	1
One twist	2	2
1 ½ twists	1	1
Two twists	2	2

Cutting Möbius Bands Activity

The next activity will involve cutting the Möbius Band in half along the line down the middle. This should not be accomplished by cutting in to the band from the side - the cut needs to run along the line. For safety reasons, and to discourage students from stabbing the scissors through the strip, they will be shown how to pinch one end of the band together and make a single cut, which if they open it out will give them a place to start cutting from. If you see anyone ignoring this or stabbing through the paper, encourage them to use this method.

Students should cut all the way along their middle line (or down the middle, if their line veers significantly off course). If students have not done this activity before, they might find it amazing that the shape does not break into two separate pieces - but if they have seen it before, you should encourage them not to spoil the surprise for people around them.

They will also need to fill in the table to record how many faces and edges this shape has. It is equivalent to a Möbius Band with one full twist, so it should have two edges and two faces.

Next the students will be asked to mark up their final two paper strips with lines splitting them into thirds and quarters lengthways. To mark thirds, they will need to use their ruler to split the 3cm width into three 1cm sections, which can be eyeballed if time is short, and draw in the lines on both sides of the paper. Quarters is easier as it can be achieved by folding the paper in half and in half again, but the students should make sure they draw in the lines again on both sides to make sure they can be clearly seen. Then these strips will be made into (half-twist) Möbius Bands to cut along the lines, again using the safe starting technique.

These strips will split into two linked pieces, in two different ways. If students finish quickly, encourage them to think about why this has happened, and what shapes they now have. Ask them to compare the lengths of the strips.

These strips will not be very wide, and when being cut up will be likely to rip or break unless the students are very careful. If one rips, it may not be possible to reattach it without altering the number of twists, so it's better for students to start again if this happens.

To investigate the number of edges and faces, students will be shown on a slide to split the two bottom rows into more boxes, so they can then write in the information for the two pieces separately.

½, cut in two	2		2	
½, cut in three	1	2	1	2
½, cut in four	2	2	2	2

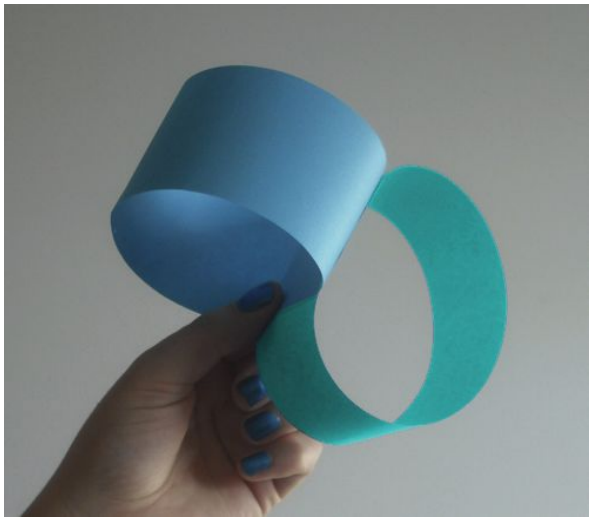
Square and Hearts Cutting Activity

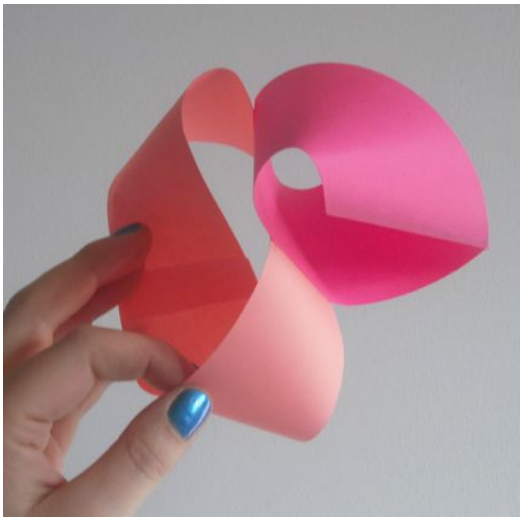
For a final activity, the students will be shown two more shapes you can make using Möbius Bands - a square, and a pair of hearts. However, the identity of the shapes you get should not be revealed ahead of time!

Students will be given four strips of paper in two colours, and should use one pair of strips to make two no-twist Möbius Bands, one in each colour, and the other pair to make two (half-twist) Möbius Bands.

Crucially, the two Möbius Bands should twist in opposite directions - if they start with an untwisted strip and turn one side over towards themselves to make one band, they should make the second by twisting the same side over away from themselves, to make sure the orientations are different. It's difficult to look at two bands that may or may not be the same and judge by looking (although if they are the same, you should be able to hold them up so they look like the same object - if you think they're different, it may just be that you're holding them differently so be careful!). If you're not sure, bring the students more paper so they can make another, so they definitely have two different ones.

They will then be asked to stick them together in pairs at right angles to make the two objects shown below.





Cutting down the middle of each of the two strips will give a square, and a pair of linked hearts respectively. Students will be given time before they do this to try to think about what will happen - you should encourage them not to start cutting until they've thought about it for a little while. When they finish, get them to think about why these shapes happen. Why do you get four right-angle corners? Where are these four corners on the heart shapes? Why do they form hearts in the Möbius case?