Möbius Bands Masterclass: Extra Background Information

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August Möbius

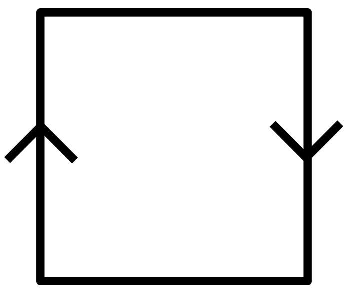
August Ferdinand Möbius was born in Germany in 1790, and trained as an astronomer under mathematicians and astronomers including Karl Mollwiede, Johann Pfaff and Carl Friedrich Gauss. He also studied mathematics, and is famous among topologists for the Möbius Band which bears his name, but also had interests in other areas of mathematics too, including geometry and number theory. For example, the Möbius Function describes the different factors a number has, and is 0, 1 or -1 depending on how the number is made up.

Möbius was appointed a professor at the University of Leipzig, where he lived until the end of his life in 1868. He had a son named Theodor, who was a philologist and studied Norse literature.

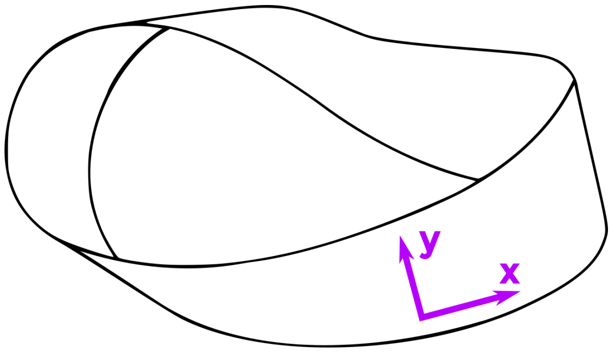
The Möbius Band

The Möbius Band was discovered in 1858, by Möbius and also at the same time, independently by another German mathematician, Johann Listing. Listing discovered the Band at around the same time, but spent more time investigating the properties of bands with higher numbers of twists than Möbius did. If they had decided to call it a Listing Band instead, it might have been a more appropriate name, as the band does 'list' (lean) to one side all the way around its length!

The Möbius Band is sometimes also referred to as a **Möbius Loop**, or a **Möbius Strip**. It is a shape which only has one side, and only has one outside edge. This diagram shows the 'matching instructions' for making a Möbius band - you need to glue the two marked edges together, but in such a way that the two arrows point in the same direction. This diagram shows a square, but in practise a longer rectangle or strip is needed to be able to make the ends meet!



The interesting behaviour of the Möbius Band observed in this workshop is caused by these two properties - having only one side means you can draw a line between any two points on the band without crossing over the edge, and having only one edge means if you cut it in half it will not split into two pieces.



The Möbius Band is described as being **non-orientable** - this means that it's not possible to define which way is up, consistently. If you draw a pair of coordinate axes somewhere on the band, so that x runs along the strip and y points upwards, and then move these axes along, even if you keep x pointing in the same direction all the time, you can move it far enough that when you get back to the start, y is now pointing the other way, because of the twist in the band. So the orientation is not consistent. The property of being non-orientable is one of the things that gives the Möbius band its interesting properties.

Another nice property of this object is that it is **chiral** - it has a handedness, and it's possible to make left-handed and right-handed Möbius bands, depending on which way you twist the paper when making the band. The two types of band are used in the hearts activity in the workshop, and while they look and behave similarly, they are two different objects. Looking at two bands, if they are the same handedness they can be positioned to look the same, but if they're different, it's harder to tell as they might just not be in the right orientation.

Other objects with this chiral property include your hands - they look similar and have symmetry, but you can't orient your two hands so they look the same - you'll always find you're looking at two different shapes, or the front of one and the back of another. There are also certain types of molecules in chemistry which come in two different chiral forms, an L-isomer and and R-isomer. Sometimes the two different forms of the same molecule can behave very differently - for example, menthol is used in toothpaste to give it a minty taste, but its isomer, made from the same atoms in a mirror molecule, tastes musty and bitter.

Applications

Möbius bands have some real-world applications - they're often used as drive belts, because they wear evenly on both sides. So conveyor belts, drive belts, data storage tapes and typewriter ink ribbon can all be attached in a Möbius configuration to create a longer-lasting product, which uses both sides of the tape without having to turn it over.

You can also use a Möbius design for a rollercoaster - the Grand National in Blackpool is one of three in the world. This type of rollercoaster has two tracks, which run alongside each other with two cars running at the same time, but the ride includes a point where the tracks cross over each other, so the cars arrive back at the original station and seem to have switched tracks!

Grand National rollercoaster, Blackpool (image by Wikipedia user WillMcC)

The Möbius band is also used as part of a magic trick, which has been performed by several stage magicians and was very popular in the first half of the twentieth century. The trick involves cutting three strips of paper or fabric in half, but each has been secretly twisted in a different way and produces a different result.

It's also possible to knit a Möbius scarf, which has a twist and connects around in a loop. It can be worn folded flat at

the front, as the twist allows it to join without part of it sticking up. (Möbius Muffler: image from mathsgear.co.uk)

Topology

Topology is the study of mathematical shapes, and is concerned with how things connect together. The Möbius Band is of great interest to topologists and its interesting properties mean it's a nice example of a topological shape.

One thing topologists use to tell different shapes apart is topological invariants - these are properties which your shape has which might not be true of other different shapes, and so if you can show that an invariant is different for two shapes they must be different shapes (but if the invariant is the same, that doesn't necessarily mean they are the same shape!) For example, you could count how many edges something has.

A cylinder has two edges, so it's definitely different to a Möbius Band, which only has one edge, but a circle also only has one edge, so this doesn't help us to tell it apart from a Möbius Band - so we'd have to use a different invariant, like the number of holes it has through it. The process used in the workshop, or drawing a line down the middle and edge of the band, means we're using the invariant of the number of sides and edges to tell apart the objects with different numbers of twists. However, it doesn't distinguish between the shapes with zero, two and four half-twists, nor between the 1 and 3 half-twist cases.

One other property of the Möbius strip which makes it different from a flat shape like a circle is that it can be coloured in differently. **The Four Colour theorem** says that for any way you can divide up a flat piece of paper into regions, you can always colour it using four or fewer colours, so that no two regions that share an edge are the same colour (try it now!). On a Möbius Band, it's possible to split up the surface so that six different colours are needed - all six regions each touch all of the other five regions.