The Royal Institution
Science Lives Here
Masterclass network

Kantor Ri Primary Mathematics Masterclasses

OTS Masterclass Patterns in Primes

Session Leader Notes

Inspiration:

An exploration of prime numbers and factor properties, leading to drawing generalisations about properties of prime numbers (and maybe even an attempt to predict primes!) drawn from talks by Dr. Vicky Neale and the research and thinking underpinning her book 'Closing the Gap'.

Overview of Activities:

- Starter on finding factors of any number
- Using multilink cubes to visually demonstrate factors
- Exploring patterns of primes in a 36 number grid
- Optional: Watch Vicky Neale YouTube video and discuss Guardian article about largest prime found
- Expanding the pattern – finding primes up to 100 using the Sieve of Erastosthenes method
- Expand even further – can we find even more?
- Finish with Marcus de Sautoy video on using primes to keep the internet safe
- Optional extension activities

General Masterclass resources needed:

- Register of children
- Consent forms and emergency information to hand
- Stickers and markers for name badges
- Adult register
- Ri child protection policy
- Paper and pencils/whiteboards for workings
- 2 different coloured post-it notepads
- Settling activity provided in slides
- Drinks and biscuits

Specific resources needed (available on slides or separately in worksheet folder):

- Enough multilink cubes for approx. 100 per number square (if there is not enough for this, groups can start off by themselves with a few cubes and then collect together in larger groups and build a complete model together)
- Copies of Worksheets:
  - **Worksheet 1.** Large 36 number square (1 per pair, or 1 per group of 3/4)
  - **Worksheet 2**. Small 36 number square (cut each printout into 4 and give 1 grid per student).
  - **Worksheet 3.** 100 number square (cut each A4 printout into 2 and give 1 grid per pair of students)
  - **Worksheet 4.** 324 number square (one per student so they can take home if they don't finish it in class)
- Optional extra: Print student takeaway with guidance for extension activities following this masterclass (one per student)

Support resources:

- Helper notes: An overview of the Masterclass content and activities
- Extra information and background on primes and the specific examples used in this Masterclass can be found in YouTube videos by Vicky Neale, Marcus de Sautoy and Numberphile – references and links included in the slides. Watch these in advance of delivering the Masterclass and also look at the Guardian article (2018) referenced in the presentation.

Things to prepare in advance

- Print worksheets and resources as detailed above
- Gather pencils needed for shading (coloured pencils would be best)

- Gather general Masterclass resources
- Check that you can access and run YouTube with volume on.

Ask the Ri

Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk

Feedback

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

Time plan of Masterclass:

Slides & Time	**Overview**	**Activity** (*Session leaders are welcome to choose ways to explore the topic that suits them best*)
Slides 1	Patterns in Primes 'Getting warmed up' Instructions on screen.	Warming up exercise while waiting for session to start - Helpers and Speaker circulating and chatting with students Students work independently or together, using their own methods
Slides 2-7 5 minutes (5)	Introduction to the Ri *[Only include these slides for the first session in the series – otherwise remember to hide the slides before you start the Masterclass]*	Use these slides to introduce the students to the work of the Ri and other ways they can get involved – see notes on the slides for more detail. In particular: • The Ri is a science communication charity which has been around since 1799. We've got a huge amount of history and lots of famous scientists lived and worked at the Ri. Most importantly, we've always been about communicating science to the general public – and that's something we still do today. We do talks and activities for the public as well as with schools all across the UK. • There are lots of family events at our building in London, including family fun days and holiday workshops just like the Masterclasses. • The CHRISTMAS LECTURES are for young people and are on television at Christmas time, looking at a different topic every year. We've got an archive on our website of all of the recent series plus many of the older ones. The CHRISTMAS LECTURES are what started the Masterclass programme. See slide notes for links. • We have a YouTube channel with lots of videos for people interested in science (and maths engineering, computer science…), especially our ExpeRimental series which is all about doing experiments at home. • Students are part of a big family of Masterclass attendees – we have been running Masterclasses since 1981. • Students at series running within reach of London will be invited to a Celebration Event at the Ri in June/July. • You can become an Ri Member to get more involved with what we do (and enter the ballot to buy tickets to the CHRISTMAS LECTURES filming).
Slides 8	Title slide	Patterns in Primes

Slides & Time	**Overview**	**Activity** (*Session leaders are welcome to choose ways to explore the topic that suits them best*)
Slides 9 5 minutes (10)	Getting warmed up: Solutions for the warm-up activity	Ask the students what their method was for solving. Review the definition of factor: a number that divides into another number with no remainder This sets up the next activity which is looking for prime number factors. There is a solution on slide 9 should students need help – ask them if they used a different way to solve it.
Slides 10 **20 mins for worksheet 1 activity** (30)	**<u>Worksheet 1: Place the cubes on the number grid</u>** Large 36 grid and blocks activity. *Introduce activity and get students going on it*	**** GIVE OUT WORKSHEET 1 AND MULTILINK CUBES**** One worksheet between pairs or groups of 3/4 One cube represents one unique factor. For each number on the grid, build a tower with the correct number of cubes – one for each unique factor. Explain this to the group and talk them through a few examples e.g… 1 has only 1 factor – therefore one cube 2 and 3 have two factors each – therefore two cubes on each square. Ask them how many would be in the tower for 8 (answer is 4 cubes). When students understand the concept, they should work together and help each other create a complete model of factors by placing the cube towers on the number grid. *(If you are running a shorter session and/or have too few cubes, make the groups work together. You can assign rows to each student to work on so they do it as efficiently as possible).* *They may spot patterns at this point – those who finish quickly can be prompted to study their 'city of towers' for patterns.*
Slides 11–13	**<u>Worksheet 1:</u>** **<u>Questions to think about</u>**	• **What interesting patterns can you spot on your grid?** – *Students explore patterns they see (might see patterns relating to certain columns, or to primes or that the height of blocks appear to get bigger as numbers increase, etc.)* • **Which number has the tallest tower?** – *36 is tallest, it has largest number of factors* • **Which number has the smallest tower of factors (blocks)? Why is this?** – *1, it is only divisible by itself* • **Which numbers have the next smallest tower of factors (blocks)?** – *All the prime numbers have 2 blocks on them.* **Why is this?** - *they are divisible by themselves and 1.* • **Do all the numbers have an even number of factors (blocks)?** – *No, some are odd.* **Why is this?** - *For even ones each factor pairs add up to make an even number. Odd ones have one factor 'pair' that is a square number e.g. 25 = 5x5. The factor is only counted once, so total number of factors is odd.* • **Is there any pattern to the numbers that have only two factors?** – Don't worry if they don't spot a pattern. This question leads on to the next activity where they explore this further.

<table>
<tr><th>Slides & Time</th><th>Overview</th><th>Activity
(Session leaders are welcome to choose ways to explore the topic that suits them best)</th></tr>
<tr><td>Slide 14</td><td>Worksheet 1 SOLUTION

For students to check their work once completed activity:

Either hand out print-outs of solution to adult helpers so they can show solutions to students when they are ready or show this slide once students have completed activity.</td><td>Solution:
WORKSHEET 1: Large 36 grid SOLUTION
Number of blocks supposed to be on each square shown in red
<table>
<tr><td>1_{1}</td><td>2_{2}</td><td>3_{2}</td><td>4_{3}</td><td>5_{2}</td><td>6_{4}</td></tr>
<tr><td>7_{2}</td><td>8_{4}</td><td>9_{3}</td><td>10_{4}</td><td>11_{2}</td><td>12_{6}</td></tr>
<tr><td>13_{2}</td><td>14_{4}</td><td>15_{4}</td><td>16_{5}</td><td>17_{2}</td><td>18_{6}</td></tr>
<tr><td>19_{2}</td><td>20_{6}</td><td>21_{4}</td><td>22_{4}</td><td>23_{2}</td><td>24_{8}</td></tr>
<tr><td>25_{3}</td><td>26_{4}</td><td>27_{4}</td><td>28_{6}</td><td>29_{2}</td><td>30_{8}</td></tr>
<tr><td>31_{2}</td><td>32_{6}</td><td>33_{4}</td><td>34_{4}</td><td>35_{4}</td><td>36_{9}</td></tr>
</table></td></tr>
<tr><td>Slide 15

15 minutes for w'sheet 2 activity
(45)</td><td><u>Worksheet 2: Patterns in primes</u>
Small number grid
Introduce activity and get students going on it</td><td>**GIVE OUT WORKSHEET 2 AND COLOURED PENCILS**
Circle all the prime numbers on <u>small 36 square grid</u> (students should refer to their multicube towers or the solutions to worksheet 1 to identify quickly all the primes).
Can students now spot any patterns in the prime numbers?...</td></tr>
<tr><td>Slides 16-17</td><td><u>Worksheet 2: Patterns in primes</u>
Small number grid

Solution and things to think about</td><td>
<table>
<tr><td>1</td><td>(2)</td><td>(3)</td><td>4</td><td>(5)</td><td>6</td></tr>
<tr><td>(7)</td><td>8</td><td>9</td><td>10</td><td>(11)</td><td>12</td></tr>
<tr><td>(13)</td><td>14</td><td>15</td><td>16</td><td>(17)</td><td>18</td></tr>
<tr><td>(19)</td><td>20</td><td>21</td><td>22</td><td>(23)</td><td>24</td></tr>
<tr><td>25</td><td>26</td><td>27</td><td>28</td><td>(29)</td><td>30</td></tr>
<tr><td>(31)</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td></tr>
</table>
What patterns are created by the prime numbers?...The grid shows all prime numbers circled.
Can we create a rule for predicting prime numbers?... Not exactly, but they do all fall into two columns so we can predict this will carry on. Recognising patterns can help us to predict.
What do all the primes have in common (apart from 2)?... they are all odd.
If the prime numbers only appear in these two columns (ignoring 2 & 3), we might predict that this rule holds for larger primes. What is the relationship?...They are always one more than or one less than a multiple of 6</td></tr>
</table>

Slides & Time	Overview	Activity (*Session leaders are welcome to choose ways to explore the topic that suits them best*)
Slides 18 **10 minutes** (55)	*Optional:* Youtube video: **Dr. Vicky Neale** talking about patterns in the prime numbers "This is exactly the research of some mathematical experts!"	Watch as much of the video as your class would find interesting. The main detail for setting up the rest of the session is between 1.00-8.00 (If you need to run a shorter session remove this section) <u>Brief summary</u>. There are lots of patterns in prime numbers that are shaded on a 10 x 10 number grid. Even columns don't have any shaded (apart from '2'), due to divisibility by 2. 2 and 3 are the only consecutive numbers that are shaded. As the grid continues, the gaps between primes seem to widen, and this makes sense because larger numbers have the potential for more factors. Will prime numbers eventually stop occurring, as these gaps get larger? That's difficult to comprehend, so we use computers to help us. A theorem that has been around for a while states that 'there are infinitely many primes' and of this we are sure. Vicky explains the proof for this. Vicky's proof is quite complex, but easy to understand with her explanation. In the rest of the session we are hoping to find a rule to predict where prime numbers might occur.
Slides 19 5 minutes (**60:** *Takes you to end of first hour*)	*Optional*: Dr. Vicky Neale says 'there are infinitely many primes'	2018 Guardian article: Finding the next prime numbers… Introduce this concept of 'infinitely many primes' Discuss the news article about the recently discovered prime number, largest known one at time of article (and still the case in 2019).
Slide 20 **10 mins** (70)	**BREAK**	Refreshment and comfort break, if doing a longer session
The next section uses a mathematical sieve technique named after the Greek mathematician Eratosthenes. This effectively is a technique to 'sift out' the non-prime numbers easily so you are left with only the primes. You therefore focus on finding patterns for multiples in number grids (e.g. even numbers are in columns and multiples of 3 have a diagonal pattern in a 100 grid). Spotting patterns like this is a fundamental skill in mathematics….		
Slide 21 **20 minutes for w'sheet 3 activity** (90)	**<u>Extending the pattern.</u>** *Introducing the activity for worksheet 3.*	How can we 'discover' new prime numbers? introduce the concept that it would be very difficult to identify all primes in a larger number grid.
Slides 22-24	**<u>Worksheet 3. Extending the pattern.</u>** *Introducing Erastosthenes Seive and the activity for worksheet 3.*	Eratosthenes Sieve is one of the oldest methods for finding prime numbers, named after ancient Greek scientist who invented the method. It allows students to try and discover primes for bigger number squares. Here, we have shown a 100 number square. Instructions are on slides – explain the process before handing out the worksheets. ****GIVE OUT SHEET 03 (100 SQUARE ERATOSTHENES SIEVE) **** Students should be encouraged to look for patterns of multiples on the grid that are of interest and allow them to 'sieve out' each multiple with relative ease (e.g. 2 and 5 multiples are in columns, 3s are diagonal).

<table>
<tr><th>Slides & Time</th><th>Overview</th><th>Activity
(Session leaders are welcome to choose ways to explore the topic that suits them best)</th></tr>
<tr><td rowspan="2">Slides 25,26</td><td><u>Worksheet 3. Extending the pattern…</u>

Key questions to think about.</td><td>ERATOSTHENES SIEVE cont…

Questions to think about displayed on screen and below.
NOTE: answers are *animated* in the PPT presentation, so will appear when you click to move on.

Give students time to talk about them and feedback to the whole class.
Other things you could discuss:
Was this a good method to use?
Is there a better one?
How are primes being discovered right now?</td></tr>
<tr><td><u>Worksheet 3. Extending the pattern…</u>

SOLUTIONS for 100 number grid</td><td>There are 25 prime numbers between 1 and 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

• Which numbers get crossed out more than once, and why? Numbers with multiple prime factors

• Which numbers don't get crossed out at all, and why? Prime numbers

• What pattern on the grid did the 3 times table have? Diagonal pattern

• Why do you not need to test for 4, 6 or any number above 7 in this grid? Looking at numbers on the top row: 4, 6, 8, 9 and 10 are multiples of 2 or 3 so have already been sieved. In other words, <u>we only need to sieve for prime numbers</u>. Also, If you chose a number greater than 10 (square root of max number on grid), its factor pair must be less than 10 and has therefore already been sieved.</td></tr>
<tr><td>Slide 27</td><td>Optional:

Prime numbers up to 120 Animated Gif showing Sieve of Eratosthenes solution</td><td>This gif is a pleasing way to animate the 100 square activity the students have just done (this gif goes up to 120).

Students normally enjoy watching this, but it does skip through the numbers very fast – it's worth explaining initially that you will leave the gif running to watch it loop round a few times. They should realise that they are seeing the activity they have just completed after a few cycles. If they didn't discover some of the patterns (such as multiples of 3 going diagonally), use this gif to illustrate it to them now.</td></tr>
<tr><td>Slide 28-32

20 minutes for w'sheet 4 activity (110)</td><td><u>Let's go bigger</u>

Lets choose a number grid

Introducing worksheet 4.</td><td>These slides introduce the next activity of finding even bigger primes using a larger number grid. The students are presented with 4 number grids to choose from – which one is easiest to sieve?

Excluding 17x17 and 19x19 is easy because these don't have multiples of 2 in columns. That leaves 18x18 and 20x20 to choose from:

Many students will say 20x20 grid (multiples of 2 and 5 line up in columns so easy to sieve), but you can show them that an 18x18 grid is better because multiples of 2 and 3 line up in columns and so are easy to sieve – that leaves fewer remaining numbers to sieve in 18x18 grid once they've done these.

Also of note for 18x18 grid: multiples of 5 are easier to sieve than first appear! Students should be able to identify the 'one bigger than or one less than multiples of 6' rule for primes with this grid. These will be explored a little in next set of slides.</td></tr>
</table>

Slides & Time	**Overview**	**Activity** (*Session leaders are welcome to choose ways to explore the topic that suits them best*)
Slides 33-34	**Worksheet 4: 324 grid**	****GIVE OUT SHEET 04 (324 SQUARE ERATOSTHENES SIEVE)**** 1 per student Remind the students of the first steps of the sieving process with these slides. Get them to quickly sieve for 2 and 3. Pause and look at the pattern in the grid that shows you can see the 'one above or one below multiple of 6 rules' in this grid in the same way as you could see it in the 36 grid
Slides 35-39	**Worksheet 4: 324 grid** *Cont'*	Bring the students back to look at the presentation when they have all sieved for 2 and 3. Go through this set of slides with them to discuss completing the sieve before leaving them to do it independently. *If they don't have enough time to complete, stop around 5 minutes before end of session. Tell them they can take these home to complete.*
Slides 40-42	**Worksheet 4: 324 grid** SOLUTIONS	There are 66 prime numbers between 1 and 324: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317 *If they've run out of time and are taking home to complete, tell them to do an internet search for 'prime numbers to 500' when they get home to see solution again.*
Slide 43 **10 minutes For video and recap /round-up** (120)	**Why bother with prime numbers?** Marcus Du Sautoy – short video	We have managed to predict where primes might occur (in the column one smaller or one greater than a multiple of 6) however, this doesn't help us predict where the next prime will occur. As Vicky said, powerful computers are doing this for us. Why might we want to know 'new' prime numbers? Marcus Du Sautoy helps to explain why 'new' primes are useful. The fact that they are unpredictable forms the basis for online cryptography. That these are not predictable means that our data and financial information is stored safely. Large prime factors are used as a sort of lock-and-key system for our data, that only the website you are using knows the answer to. If they did become predictable, this data might be at risk of being hacked!
Slide 44, 45	**Recap** Feedback, reflection time, questions time	Recap points are animated on slide – each one appears when you advance presentation 'Optional Extra' activities are available for those students wanting to investigate further (provide printout) Any further questions?
Slide 46	Optional extension: Visualising factors *Could replace the 324 grid activity with this if you have less time*	Another way to visualise factors and primes is shown in this video. Ask the students which shape is used to represent primes (circles), and why they might think that shape was chosen. You could also ask them to discuss why specific shapes such as squares and triangles were chosen to order non-prime numbers. Optional activity using multilink cubes: Get the students to have a go at this themselves by choosing a non-prime number: count that number of blocks and create their wen pattern to represent the factors (individually or in pairs). Get them to explain why they chose their pattern
Slide 47	Tidy up and questions time	Ask the Ri: Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk