**Magic Squares Masterclass**

Thank you very much for leading this Masterclass. We hope that you enjoy working with this material as much as we enjoyed putting the activities together. We do appreciate all the effort that our volunteers put into bringing inspirational Mathematics Masterclasses to children around the country. Don’t forget that we’d love to know your thoughts on the Masterclass – more detail at the end of this section.

**Inspiration for this topic:**

An exploration of number bonds and symmetry, magic squares capture something of the beauty and joy of manipulating numbers. They have a long and distinguished history, having fascinated mathematicians for centuries.

**Overview of Activities:**

This Masterclass begins with a challenge to make a 3x3 magic square from the numbers 1-9.

* It will then explore some of the history of magic squares, using three specific examples:
* the magic square carved into the façade of the Sagrada Familia in Barcelona
* the magic square depicted in the Durer painting “Melancolia”
* the ancient Lo Shu Square from China
* It will then allow the students to explore the structure of magic squares by constructing their own examples (Random Total Magic Square (YouTube) and Date Magic Square (NRICH))

**General Masterclass resources needed:**

* Register of children
* Consent forms and emergency information to hand
* Stickers and markers for name badges
* Adult register
* Ri child protection policy
* Paper and pencils/whiteboards for workings
* 2 different coloured post it note pads
* Settling activity if not included in specific Masterclass (this one has magic square activity)
* Drinks and biscuits

**Specific resources needed:**

1. 3x3 magic square digit cards – digits 1-9 (use one set between 2 students as an introduction activity). For a variation with digits 2-10, draw a zero beside the 1. You will need to pre=-prepare these or should have scissors for the students to cut the digit cards up. *Download: two sets per sheet.*
2. Durer magic square – one each. *Download: six per sheet.*
3. 4x4 magic squares worksheet – one each. *Download: two per sheet.*
4. Sagrada Familia magic square – one each. *Download: six per sheet.*
5. Random total magic square worksheet – one each. *Download: one per sheet.*
6. Special date magic square worksheet – one or two each as extension/takeaway. NB: You can also print the session leader/helpers worked example for the students to take home. *Download: one per sheet.*
7. OPTIONAL Additional nrich activities connected to this Masterclass. *Print or show on-screen.*

**Support resources:**

* Helper notes: An overview of the Masterclass content and activities
* Additional information: Extra background on magic squares and the specific examples used in this Masterclass
* Worked example: Special date magic square. The answer will change for each date you pick, so here is a worked example to help guide you, the helpers and the students.

**Things to prepare in advance**

* Print worksheets and resources as detailed above
* Gather general Masterclass resources
* Check that you can access and run YouTube or numberphile video
* Work out example magic square for the date of the Masterclass (<http://nrich.maths.org/1380>) or you can use the provided worked example for 6th April 2018

**Ask the Ri**

Don’t forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: [masterclasses@ri.ac.uk](mailto:masterclasses@ri.ac.uk)

**Feedback**

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

**Time plan of Masterclass:**

| **Slides & Time** | **Overview** | **Activity** |
| --- | --- | --- |
| Slides 1,2,3  10 minutes (10) | Introduction  Instructions on screen.  Helpers and Speaker circulating and chatting with students | \*GIVE OUT DIGIT CARDS (01) AND SCISSORS AND ROUGH PAPER/PENCILS OR MINI WHITEBOARDS\*  Set the scene by asking the students if they know what a magic square is.  If not introduce them to the idea that it is a grid of numbers where the total of each row, column and diagonal add to the same total (The “Magic” total). Explain that we are going to enjoy exploring magic squares, their long history, and their amazing mathematical properties!  First activity: Working in pairs, students make a magic square (3x3) total of columns, rows and diagonals are all 15. Encourage them to experiment and make mistakes, and gradually improve their answers until they have a complete solution.  When someone has a solution ask them to write it on the whiteboard/flipchart (so you can build up a set of solutions)  If they have found a solution can they find a different one? How many different squares can they make? Ask them to add solutions to whiteboard. How many solutions are there? (The answer is essentially 1, but others are transformations of one another; discussion of rotation and reflection) There are 8 versions of this unique solution, obtained in this way.  When you have built up a few “different” solutions then break to introduce the Ri, if appropriate, or jump to slide 13 |
| Slides 4-9  5  minutes (15) | Introduction to the Ri  *[Only include these slides for the first session in the series – otherwise remember to hide the slides before you start the Masterclass]* | Use these slides to introduce the students to the work of the Ri and other ways they can get involved – see notes on the slides for more detail. In particular:   * The Ri is a science communication charity which has been around since 1799. We’ve got a huge amount of history and lots of famous scientists lived and worked at the Ri. Most importantly, we’ve always been about communicating science to the general public – and that’s something we still do today. We do talks and activities for the public as well as with schools all across the UK. * There are lots of family events at our building in London, including family fun days and holiday workshops just like the Masterclasses. * The CHRISTMAS LECTURES are for young people and are on television at Christmas time, looking at a different topic every year. We’ve got an archive on our website of all of the recent series plus many of the older ones. The CHRISTMAS LECTURES are what started the Masterclass programme. See slide notes for links. * We have a YouTube channel with lots of videos for people interested in science (and maths engineering, computer science…), especially our ExpeRimental series which is all about doing experiments at home. * Students are part of a big family of Masterclass attendees – we have been running Masterclasses since 1981. * Students at series running within reach of London will be invited to a Celebration Event at the Ri in June/July. * You can become an Ri Member to get more involved with what we do (and enter the ballot to buy tickets to the CHRISTMAS LECTURES filming). |
| Slides 10,11,12 | Repeat of intro slides. | See notes on slides 1-3  Can start from here if more convenient. |
| Slides 13,14  15  minutes (30) | Discussion of introductory work  Some additional challenges are on slide 14, which can be used with the whole class, or for students finishing the initial challenge quickly  Answers appear in red, slide 14 is animated | Slide 13 shows the eight “different” solutions possible with numbers 1-9  Lead a discussion about whether these are indeed different: what do we mean by different, or the same?  *Conclusion that there is essentially only one 3x3 magic square, can be transformed by rotation and reflection.*  **The slide is animated so that on the click the question about the link between the pairs of squares appears**.  *Concept of complementary magic square. Some children see the link as transformation. Another way is n maps to 10-n in each square.*  Other challenges:   1. Work on 2-10 square (by changing the “1” card to say “10”) (Ans: The magic total will now be 18) 2. Try to make a magic total of 21(Ans: use the numbers 3-11) 3. Try to make a magic total of 150 (Ans: make all numbers 10 x bigger, so 10,20,30..90) 4. Try to make a magic total of 45 (Ans: make all numbers 3 x bigger, so 3,6,9…30) 5. Can you make a magic total of 16? (Not if using consecutive whole numbers: the magic total then will always be a multiple of 3)   T**he answers to these questions appear on the slide on consecutive clicks** |
| Slide 15  5 mins (35) | Magic square in history Lo Shu | Show the students the images of historic magic squares. Hopefully they recognise the magic square 1-9, magic total of 15, which they have been working on.  The first magic square is the Lo Shu Square, and is from China.The picture of the turtle relates to the Lo Shu square, and has an associated myth:  In ancient China there was a huge [deluge](https://en.wikipedia.org/wiki/Flood_myth): the people offered sacrifices to the god of one of the flooding rivers, the Luo river, to try to calm his anger. A magical [turtle](https://en.wikipedia.org/wiki/Turtle) emerged from the water with the curious and decidedly unnatural (for a turtle shell) pattern in its back. You may recognise that pattern… |
| Slides 16,17,18  10 minutes (45) | Introduce idea of 4x4 magic square via Melancolia  There are an amazing number of ways of making 34!  Answers appear in red, slide 18 is animated | Tell the students we are now moving onto think about the fact that magic squares could be larger…  Sometimes these squares appear in art. Slide 16 gives some background information about Durer.  Slide 17 has a larger version of the picture. Ask the students to look at the picture and to tell you mathematical images they can see in the picture.  For example: Magic square (first occurrence in Western art); compasses; 3d polyhedron (probably truncated rhombohedron =stretched cube with corners cut off (!)); spherical ball; hour glass; balance; sundial.  \*GIVE OUT COPY OF DURER MAGIC SQUARE (02)\*   * Display slide 18. Look more closely at the magic square. What do you notice? What is the total? (34 (always total for 4x4 square made using 1-16): **answer** **appears on first click**) * What year do you think this was painted in? (Remind them of Durer’s dates)(1514, at bottom! **Answer appears on second click)** |
| Slide 19,20,21  10 mins  (55) | Examples of 4x4 magic squares  Answers appear in red, slides 20,21 are animated | \*GIVE OUT COPY OF FILL IN MAGIC SQUARES (03)\*  Allow time for the students to complete these two examples. The first has a unique solution, and is easier than the second, which has two possible solutions. Copy of questions on slide 19  Answers on slide 20 (First example) and slide 21 (Second example and extension question) **Each new number appears on the next click**  Ask the children to notice that the two 4x4 magic squares here are not reflections or rotations of one another.  Extension challenge: Make your own 4x4 magic square using 1-16. Magic total must be 34  Unlike the 3x3 1-9 magic square, the 4x4 1-16 magic square solution is not unique! (There are 880 separate solutions actually! How crazy is that?? A good example of maths not always being intuitive, and sometimes giving very surprising results) |
| 10 mins (65) | BREAK | Drinks and biscuits and comfort break |
| Slides 22,23, 24,25  10 minutes (75)  slides 22,23,24 are animated | Another example of a real magic square.  Look at it in detail, for itself  Then discover the amazing link between Durer and SF squares! | Slide 22 is a picture of the passion façade of the cathedral in Barcelona, Spain, which is still under construction. It was designed by Gaudi and construction started in 1882. The plan is for it to be complete by 2026, which will be 100 years since Gaudi died.  **On the first click a circle appears**, which is a ring round the magic square sculpted into the façade. A close up appears on the next slide. The sculptor’s name was Subirachs  Slide 23: \*GIVE OUT COPY OF SAGRADA FAMILIA MAGIC SQUARE (04)\*  What is magic total? (Answer: 33 **appears on first click**)  Explain that this magic square appears on the Passion façade of the cathedral. It is telling the Bible story of the death and resurrection of Jesus.   * Why did the sculptor make the magic total 33? (Answer: because Jesus was 33 when he died) * How does the square differ from the magic squares with total of 34? (Answer: 4 numbers have been reduced by one; there is a changed number in each column, row and diagonal   Students can now compare the Durer square with the Barcelona one.  Collect ideas about what is the same, what is different.  Answer: Subirachs’ square consists of many of these same features as Dürer's. However, Subirachs chose to create a square with the magic sum of 33. To fashion such a square, four of the numbers are reduced by one. The numbers 12 and 16 become 11 and 15, while 11 and 15 become 10 and 14. Numbers 12 and 16 are therefore missing from the square, while 10 and 14 have become duplicated. Having made these alterations Subirachs then rotated Dürer's magic square through 180°. Another “wow” moment for the Masterclass!  This is illustrated, **with an animation, on slide 24.** The animation shows the rotation of Durer’s square through 180 degrees, and then by comparing the numbers in corresponding positions it is clear that just 4 numbers have been changed, but the arrangement of the rows and columns is identical.  The conclusion is made very clear on slide 25 |
| Slides 26,27  20 minutes (95) | Impressive “trick” to construct magic square with any given total. Let numberphile take the strain, or learn how to do it yourself, and try it out on the kids (I did this, it was great!!) | Demonstrate or use YouTube numberphile <https://www.youtube.com/watch?v=aQxCnmhqZko>  Show how this is done and let pupils construct their own: this means they need to choose a value for n and substitute it. Some will grasp this quickly, but some may need lots of help.  \*GIVE OUT COPY OF WORK SHEET TO MAKE THEIR OWN MAGIC SQUARE (EXTENSION/TAKEAWAY ACTIVITY) (05)\*  Asking them to fill in “n=” first helps.  Also recommend n is greater than 21 to avoid negative numbers….  Although using a smaller value could challenge those who complete the first task quickly.  Encourage them to answer the question at the bottom of the sheet, using full sentences. (Answer: the magic total is always n because if you add the totals in any column or row the number being subtracted from n is equal to the sum of the other 3 numbers.) |
| Slide 28  15 minutes  (110)  Steps appear on each click, slide 28 is animated | Concluding activity: This is a lovely take home activity if there isn’t time in the Masterclass | Finish by making a special magic square for today’s date, or for the worked example date of 6 April 2018  \*GIVE OUT COPY OF WORK SHEET TO MAKE A SPECIAL DATE MAGIC SQUARE (EXTENSION/TAKEAWAY ACTIVITY) (06)\*  Take them through the algebra for today’s date, or the worked example. **The steps appear in turn using an animation.** It is also useful to complete the example on a whiteboard/flipchart as you go through the steps so they can follow along. (If you want to encourage them to work with you, you might need to give them another copy to do their birthday, or other special day on.)  Now let them do one for their birthday, or as a present. (6th April 2018 was chosen because it was for a wedding present for a friend: nice and probably unique(!) thing to include with a card or gift)  \*CONSIDER WHETHER TO GIVE OUT A SECOND COPY OF WORK SHEET TO MAKE A SPECIAL DATE MAGIC SQUARE (EXTENSION/TAKEAWAY ACTIVITY) (06)\*  In addition you could look at NRICH example for how it works. (<http://nrich.maths.org/1380>) |
| Slide 29  10 minutes  (120) | Feedback, tidy up, questions time  Ask the Ri | Don’t forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: [masterclasses@ri.ac.uk](mailto:masterclasses@ri.ac.uk)  We will send you answers as soon as possible. Then these can be reported back to the children at their next Masterclass session. In this way you cannot be “caught out” by a question. It also demonstrates the point that not everything in maths is known, but some questions need time and research to find answers sometimes, and sometimes the answer has not been found by anyone yet, of course! Maybe our Masterclass students will be the ones who solve the problem when they are older? |
| Slide 30 | Further activities | Possible NRICH problems related to magic squares – use as extension activities or for them to do at home:  <http://nrich.maths.org/6215> Different magic square  <http://nrich.maths.org/87> Magic constants  <http://nrich.maths.org/1205> Domino magic rectangle  This slide could be printed and given out as a nice “take home” activity suggestion. |

Slides 31-36 are copies of the resources needed (also available as separate worksheets)