

# Digital Computers Masterclass

**Thanks for helping with this Masterclass session! Your support is much appreciated.**

The session leader should be able to tell you more about the content of the session, and exactly how they'd like you to help, but this sheet should give you some basic information you may find useful. If any of this seems obvious to you, that's great!

In general, for Masterclass sessions:

- While the session leader is talking to the group, don't interrupt them or distract the students unless something is wrong that needs fixing urgently. You should also watch and pay attention to what they're saying, to set a good example.
- If things need handing out to the students, wait for the session leader to signal you to do this, as it can distract the students if you start to hand things out before they're ready.
- If the students are given a task to work on, you should circulate the room to talk to the students. Wait until they've had a chance to tackle the problem before you interrupt them, and encourage anyone who looks like they haven't started yet.
- Try not to give away the answers to the students, especially if they're working on the problem and about to discover it for themselves - if they are really struggling, you can give them a hint or suggest where they might start looking.

## **In this session:**

This workshop is an exploration of binary numbers, starting from the idea of finger counting and finger-based mathematical tricks, and moving on to representing numbers in binary using your fingers (and in the standard way, using ones and zeroes).

The main activities are:

- 1. Counting on fingers**
- 2. Multiples of nine trick**
- 3. Multiplying with your fingers trick**
- 4. Mind reading trick**
- 5. Counting in binary**
- 6. Binary and decimal worksheets**

***There is a great deal more background information available on a separate sheet, if you would like more detail. Please ask the session leader if you'd like to see it.***

**Thanks again for your help with this session! If you have any other questions, please ask the session leader.**

## **In this session:**

### **Counting on fingers**

As an introductory activity, we're asking the students to think about how they count on their fingers. You can help by going around and talking to them - which of the two methods shown on the board do they use? (The methods are: starting from the thumb, or starting from the first finger). Or do they know any other methods? There's also a slide with how to count to ten in British Sign Language, which you can help the children to try. The idea is to get them thinking about counting, and how their fingers can be used to make it easier - and also, to acknowledge that there are different methods people use.

## Multiples of nine trick

The first trick using fingers is an easy method of calculating multiples of nine using your ten fingers. Hold your hands in front of yourself with your palms upwards. If you want to multiply a number between 1 and 10 by 9, simply put down the corresponding finger (counting from left to right) - then you'll find the number of fingers on either side of this gap will correspond to the two digits of the answer to the multiplication. For example, to find  $3 \times 9$ , put down the third finger along (the middle finger on your left hand). Then you have 2 to the left of it, and 7 to the right, so the answer is 27.

The students will be given some time to try this trick, and three examples put on the screen for them to try. You can talk to them about how and why this trick works - what does it mean to add 9? It's the same as adding 10 and subtracting 1, so this will increase the tens digit and decrease the units digit each time.

## Multiplying with your fingers trick

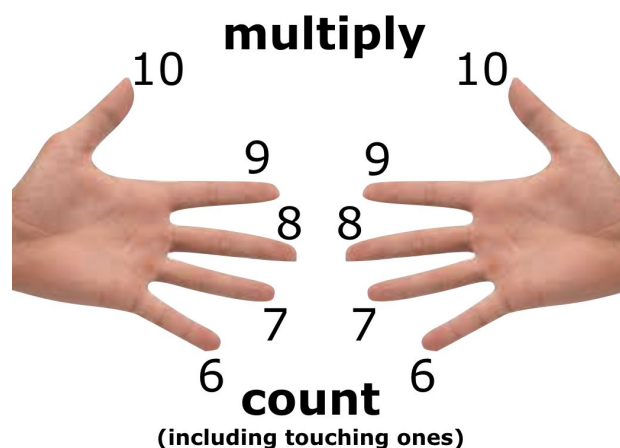
Another, less well-known trick you can use to multiply with your fingers involves numbering each of your fingers from 6 (little finger) up to 10 (thumb), on each hand. Then you can multiply as follows:

- Touch the fingers together which correspond to the two numbers you'd like to multiply, to form a 'bridge'
- Count the number of fingers below the bridge, including the bridge itself
- Count the numbers above the bridge on each side
- The result will be  $(10 \times \text{the number of fingers below the bridge, including the bridge itself}) + (\text{number of fingers above the bridge on the left} \times \text{number of fingers above the bridge on the right})$ .

The example given on the slides is  $8 \times 9 = 72$ , which involves 7 fingers in and below the bridge, and then above there are 2 and 1 fingers respectively, giving  $2 \times 1 = 2$ , and a total of 72.

Most calculations in this system work in exactly this way - although a few require a little thought. If anyone attempts  $6 \times 7$  (one of the examples on the slides), they'll find the result is  $30 + 12$  - they may be confused that the number of fingers in and below the bridge is not four, but then above there are 3 and 4, and  $3 \times 4 = 12$  which when added to 30 gives 42.

The session leader will discuss with the students that this trick is not necessarily more efficient than remembering their times tables, but it does give another way to check if you're not sure (and it's interesting that this works - it might be beyond the scope of the students to work through an algebraic proof but you may enjoy convincing yourself why it works! A proof is included in the 'Extra Information' sheet.)



## Mind reading trick

The session leader will perform a mind reading trick using a set of cards given to the students (pictured below). The cards contain the numbers 1-31, and the students will be asked to think about the date of their birthday (the day of the month, a single number between 1 and 31). They will ask the students to list all the cards that contain their birthday (e.g. "It's on cards 0 and 3"), and then correctly guess the number.

Card 4	Card 3	Card 2	Card 1	Card 0
16 17 18 19	8 9 10 11	4 5 6 7	2 3 6 7	1 3 5 7
20 21 22 23	12 13 14 15	12 13 14 15	10 11 14 15	9 11 13 15
24 25 26 27	24 25 26 27	20 21 22 23	18 19 22 23	17 19 21 23
28 29 30 31	28 29 30 31	28 29 30 31	26 27 30 31	25 27 29 31

If the student misses any cards, or misunderstands what they're being asked to do, the mind reading will fail, so it might be helpful for you to check if this happens that the student has found all the occurrences of their number - you could go over and ask them to whisper the number to you, and help them look. Occasionally they are using the month instead of the day, or looking at the numbers at the top of the cards (for example, the number 3 occurs as a number on cards 0 and 1, but not on card 3 - although it might be that they've misinterpreted the phrase 'on the card').

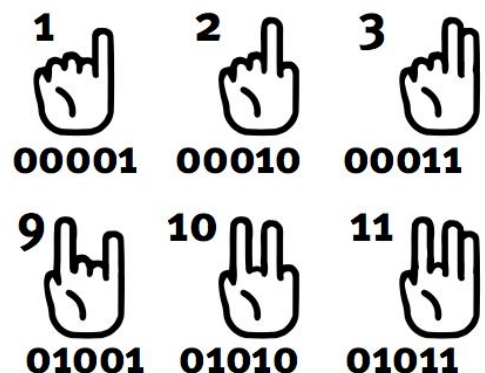
The students will then be given a chance to work out how the trick works, and will be given the hint to look at the smallest number on each card. At this stage it's probably sufficient for them to realise that if you add together these numbers on any corresponding cards, they will get the number they're looking for, although this does tie in to binary numbers.

It's probably worth just explaining if asked that the numbers have been carefully placed on the cards so that this works when you add up the top corner numbers, and they will later be asked if they see anything interesting about these numbers (to see the pattern that they're powers of two).

They will also have a chance to practise the trick for themselves - if you see anyone sitting as though they're finished, encourage them to try the trick on each other, or to guess your birthday/what number you're thinking of.

## Counting in binary

Students will now be asked to use the fingers of one hand to count in binary. They can label their fingers using strips of post-it notes, cut so they only have the sticky part, so it'll form a tube around their finger. They will need to label them 1, 2, 4, 8 and 16, and if you use the smallest finger for the smallest number it may help to remember. They can then put their fingers up and down to count upwards from 1. They should



also discover how high it's possible to count using this system on one hand (31).

Once they've done this, they can give each other the challenge of displaying a particular number. You might find they're amused by the number four, or the number three, since these both result in vaguely rude gestures, but just tell them to get on with the activity. We've suggested they write the numbers on the inside of their finger-notes so they are looking at the palm, which should make this less obvious.

You may find that if the student is using the hand they also write with, they'll need to write the numbers on before they attach the paper to their fingers, but they'll need to make sure it's on the right part of the post-it note to be showing on the front of their finger.

A 'cheat sheet' is available with images of the hands for each number, if students struggle with this activity - ask the session leader if you would like a copy.

## Binary and decimal worksheets

The students will be given some worksheets to convert numbers between decimal and binary. The first sheet has all the numbers that can be made using 1, 2 and 4 (up to 7) and then another sheet has five columns to make numbers up to 31.

The first few rows have examples and partially filled answers, but the remaining rows are blank. Students should be encouraged to fill these with their own examples - either choosing a number, then writing it in binary, or picking a pattern of 1s and 0s and working out which number it gives.

There is a further worksheet on 'writing your name in binary', which allows students to write the letters of their name as a sequence of 1s and 0s, by first converting the letter to a number using A=1, B=2 etc. This may not be used unless the session leader would like more activities, but it can be completed in the same way - except it doesn't have a column to write the binary number at the end, so instead of using the 'yes/no' format from the other worksheets, they should write 1s and 0s directly in the boxes.

### Your name in binary

Complete the table below to write your name in binary. Start from the left column and work from left to right - write 0 or 1 under each of the five columns.

Letters of your name (one per row)	Number (see table at bottom)	16	8	4	2	1

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26