Digital Computers Masterclass: Extra Background Information

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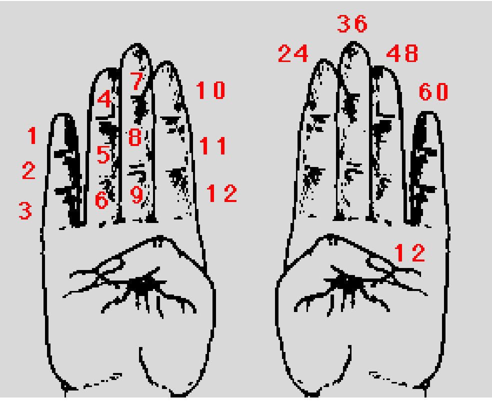
Finger counting and number systems

Historically, fingers have always been used for counting, and it is the basis of our decimal number system that a standard pair of human hands has 10 fingers. Other systems historically, including the vigesimal system used by the Ancient Mayans, have used base 20 which is a multiple of 10.

Counting on your fingers can be achieved in many ways - as well as the standard systems shown on the first slide (the 'thumb first' system is claimed to be widespread in continental Europe, while starting with the index finger is supposedly more popular in the UK and North America, although there is variation everywhere), there exist many other finger counting systems. In Japan, zero is represented by an open palm, and fingers are counted inwards for 1-5 and then out again for 6-10 on the same hand. Chinese systems also allow counting up to 10 on one hand, while counting in binary (as described in the workshop) allows you to count to over 1000 on your fingers.

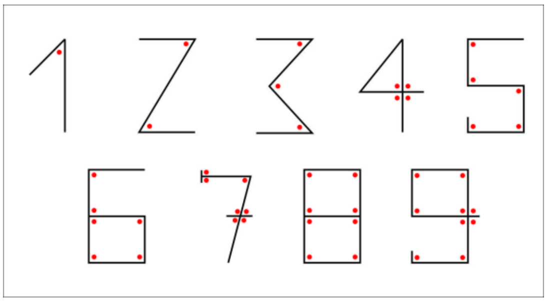


The Ancient Mesopotamians developed a system of finger counting using the left thumb to point to different parts of the right hand - 1 at the tip of the little finger, then 2 and 3 for the other sections of that finger, and 4 starting at the top of the ring finger and so on. The other hand is used to store groups of 12, up to a maximum of 60 on two hands. This is suspected to be the origin of our current 60-minute/12-hour clock system.

There are also several different systems for counting as part of sign language - British Sign Language and American Sign Language each have slightly different conventions for displaying numbers, and over 137 different languages have their own version of sign language, with variation in how they count.

As described in the workshop, early symbols used for the numbers 1, 2, and 3 consisted of one, two and three strokes representing fingers. These symbols evolved over time into the modern **arabic numerals** we use today, with the 2 symbol having two clear horizontal strokes, and the 3 symbol similarly having three parts to it.

A widely circulated myth that the number symbols each contain the number of angles corresponding to the number they represent has been debunked - it has no historical basis, and symbols have been presented in many different forms so there’s no one canonical version. Also, nobody draws a 9 like that.



Symbols for numbers have evolved over time as the number systems have changed - from simple **grouping systems**, which have a symbol for 1 and each power of the number base (in the case of decimal, 10, 100, etc) and then the number of times that symbol is represented shows how many of that power is included in the number; **place-value** or **positional** systems include a zero symbol and a symbol for each digit, and the position of the symbol within the number denotes its size. The ancient Babylonians used a place-value system, and the modern Hindu-Arabic numerals we use today date back to around 200AD.

It’s also discussed in the workshop that the word ‘five’ is similar to the word ‘fist’, consisting of five fingers, and that this is true in some other languages too, including Dutch (and languages derived from it like Afrikaans), and Polish. Numbers are part of human history and it’s no surprise they’ve formed the way language has evolved too.

More information on the history of number systems can be found in this Encyclopedia Britannica article: britannica.com/science/numeral

Mind Reading trick

Card 4	Card 3	Card 2	Card 1	Card 0
16 17 18 19	8 9 10 11	4 5 6 7	2 3 6 7	1 3 5 7
20 21 22 23	12 13 14 15	12 13 14 15	10 11 14 15	9 11 13 15
24 25 26 27	24 25 26 27	20 21 22 23	18 19 22 23	17 19 21 23
28 29 30 31	28 29 30 31	28 29 30 31	26 27 30 31	25 27 29 31

The mind reading trick presented in the workshop uses binary numbers as a basis to guess the number someone is thinking of based on its presence, or absence, on the cards. This trick has been around a long time and has many variants - with a sixth card, it’s possible to guess numbers up to 63, although 31 is a nice size to be able to guess a birthday, for which five cards is sufficient. The trick is sometimes included in Christmas Crackers as a curiosity, including instructions on how to perform the trick (although not many include an explanation of binary numbers).

The cards in our version are numbered with the power of 2 they represent, which is why the first card is labelled ‘Card 0’. On each card, numbers which contain a 1 in that position in their binary expansion are printed on the card, and for each card there are the same number of numbers, as exactly half of numbers have a 0 or 1 in each position for any given set of size 2^n. The number 0 is not included on any cards, as it is written using all zeroes. Card 0 contains only odd numbers, as these are ones which use a 1 in their binary expansion. Card 2 contains numbers which have remainder 2 or 3 on division by 4, meaning they would use 2 in their binary expansion, and similar patterns occur on each of the other cards. Card 4 contains only numbers 16 and above, as these are the ones that use 16.

Binary numbers

The system of binary uses base 2 to represent numbers, with either 0 or 1 of each power of two added together to make the number. A binary digit is referred to as one bit, a term coined by information theory pioneer Claude Shannon. Binary numbers can be added, subtracted, multiplied and divided on a per-digit basis just like decimal numbers, and binary numbers between 0 and 1 can be represented using a point, like decimal numbers - beyond the decimal point each column is ½, ¼, ⅛ and so on.

$0.01101_2 = 0.40625_{10}$

The notation of subscript numbers is used to indicate which number base the number is written in. The Ancient Egyptians used a number system for fractions based on powers of two, with symbols for ½, ¼, ⅛ etc.

Binary numbers are used by computers and all technology to store and process numbers. This is because the simple 0/1 format lends itself to transmission using simple systems like high/low voltage in a wire, or radio frequencies. Logic gates, which take two single yes/no (0/1) inputs and return a specified output, are the basis of all computer circuits and were described by George Boole in 1854, but in 1937 Claude Shannon was the first person to link logic gates with binary numbers in a way that allowed computers to use them. It’s possible to construct systems to count, add and manipulate binary numbers in many ways using combinations of logic gates in different layouts, and all computer circuits are based on this.

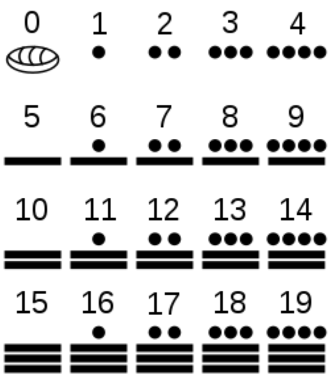
As a result, many computer-related things involve powers of two - storage media use 1024 bytes in a kilobyte (sometimes called a kibibyte to avoid confusion with the standard prefix kilo- meaning 1000) and often portable thumb-drives come in sizes like 128MB or 256MB, and the computer game Pac Man famously only has 255 levels.

Other number systems

We're familiar with the **decimal** system, in which each column of numbers represents a power of 10, and binary is just a variation of this using base 2 instead. In decimal, we use the symbols 0,1,2,3,4,5,6,7,8 and 9 - ten options for each position, whereas in binary there are two - 0 and 1.

Other number systems include **hexadecimal**, which is base 16 and uses 16 different symbols (the digits 0-9 and then the letters A-F, representing 10 up to 15). This is highly compatible with binary - a four-digit binary number represents a single digit in hexadecimal - and is therefore extensively used in computing.

Hex codes for colour use six hexadecimal digits to encode three numbers between 0 and 255 (one each for red, green and blue in the colour), and in hexadecimal ranges from 00 (none of that colour) to FF (full value). This system is used through the internet to determine colours on web pages and in software, and images taken on digital cameras are stored as a set of three values in this way for each pixel of the image. Using 256 values for each, over 16 million colour values are possible. There's a converter at hex.colorrrs.com which can be used to determine the colour of a given Hex code.



Ancient Mayans used the **vigesimal** system, or base 20 - with symbols for each of the numbers 0-19 and place value denoted by position within the number.

For their "long count" calendar, they had a slightly modified version of this system where the third column only went up to 18 and subsequent positions each represented numbers that weren't powers of 20; in this system, the entire calendar resets at 13.0.0.0.0, and this meant a reset occurred on 21st December 2012. Some believed this meant there would be an apocalypse on this date (although in Mayan culture, the resetting of the calendar was seen less as an ending and more a rebirth - so you'd be better off preparing by making new-years resolutions than cowering in a bunker).

Mayan symbols image by Bryan Derksen on Wikipedia, CC BY-SA