Curve Stitching Masterclass: Extra Background Information

The Royal Institution
Science Lives Here
Masterclass network

The activities in this Masterclass are based around the **parabola** - a curve which occurs naturally in many situations, and is an interesting shape mathematically.

Throwing a Parabola

The first activity in the workshop involves creating parabolas by throwing a ball. Any object thrown under gravity will trace out the shape of a parabola in the air. This is because gravity acts to pull the ball downwards. The speed of the ball horizontally is the same, but if it's thrown upwards gravity will first of all slow it down until it stops, then make it start moving downwards.

In reality, the shape obtained won't be a completely perfect parabola, as there will be air resistance which affects the speed the ball is moving. But if you imagine throwing a ball in a place where the only other force affecting it is gravity, the shape would be a perfect parabola.

This is something mathematicians often have to do in order to simplify calculations about real-world systems: decide which factors affecting the system will be small enough not to worry about (like air resistance) and ignore them unless they become more important - for example, if we scaled the ball up to a much larger ball, it would be more affected by air resistance due to its larger surface area. This process is called **mathematical modelling**.

Water fountains, which launch water into the air at the correct angle, will also create a parabola shape with the water - this is because each particle or droplet of water behaves like the ball, and traces out the same shape in the air.

The parabola also crops up in architecture - the shape of the curve is such that it distributes weight effectively, which means if you're designing an arch or a bridge span which will need to take a lot of weight, it's a good shape to use as it distributes the weight of the load across the structure, meaning the materials are not unevenly stressed.

The shape of a hen's egg is made up of two shapes based on a parabola - making the shell strong when compressed.

Focal points

The shape of a parabola is such that it also serves as a reflector - if a ray of light travels parallel into the parabola, the angle of the curve at the point it hits will be such that if the ray is deflected (and the angle it comes in at will be the same as the angle it's reflected at), all the light rays will be reflected onto a single point in the curve.

This point is called the **focus**, or **focal point** of the parabola, and it means that shapes based on parabolas can be used to make reflecting dishes and antennae.

If you spin a parabola around its base, you get a 3D dish shape called a **paraboloid**. This is the shape used to make satellite dishes, antennae and radio telescopes. Sound waves, light rays and signals from far away can be 'caught' by the paraboloid, and will all be focused to a point in the centre of the dish, which means a receiver can be placed there and collect all the information.

Some parks and science centres display a matching pair of paraboloids, called **'whispering dishes'**, which make use of the focal reflecting properties of the parabola. Any sound waves which travel into the paraboloid in parallel lines will all be reflected back to the focal point, in the centre of the dish where your head will be if you're standing in front of it. It will also take any sound made at this point, for example, if you speak, and beam it out as a set of parallel sound waves, towards the other dish. This means a person standing at the focus of one paraboloid can speak quietly - even whisper - and be heard perfectly clearly by someone at the focus of the other paraboloid - even if it's many metres away.

Parabolic reflectors are also used in theatre lighting - sometimes called 'par cans' - a light source, emitting light in all directions, is placed at the focal point of a paraboloid shaped mirror, which then beams the light out in parallel rays to create a strong spotlight, which can be pointed at something on stage to illuminate it.

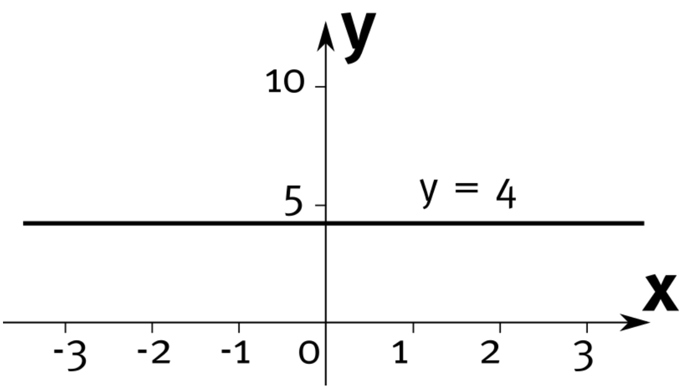
Straight and curved

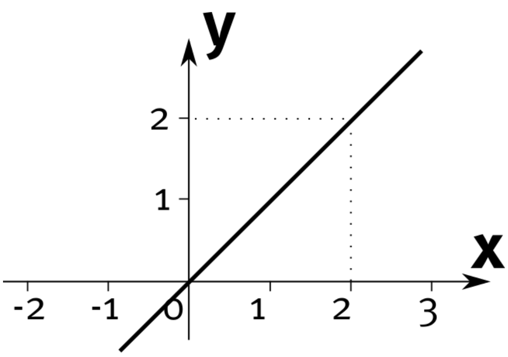
In the workshop, we use a variety of techniques to create parabolas - including folding a piece of paper. This works because the dot we draw is at the focal point of the parabola, and the parabola is made up of all the points halfway between this dot and the bottom edge of the paper. The fold lines are all called **tangent lines** to the curve - this is the line that incoming rays are reflected in, as though it were a separate flat mirrored surface at each point.

If we fold the paper at least ten times, we can start to see the curve appearing - but our curve is actually made up of straight sections of line, all joined together. To get a parabola curve with no straight sections, we would need to make a lot of folds - and for a true parabola, we would need to keep folding forever, so there are infinitely many folds. We say the parabola is the **limit** of this process - if we were able to keep doing it forever, we would get an actual parabola curve, but the more folds we do, the closer we approximate the real curve.

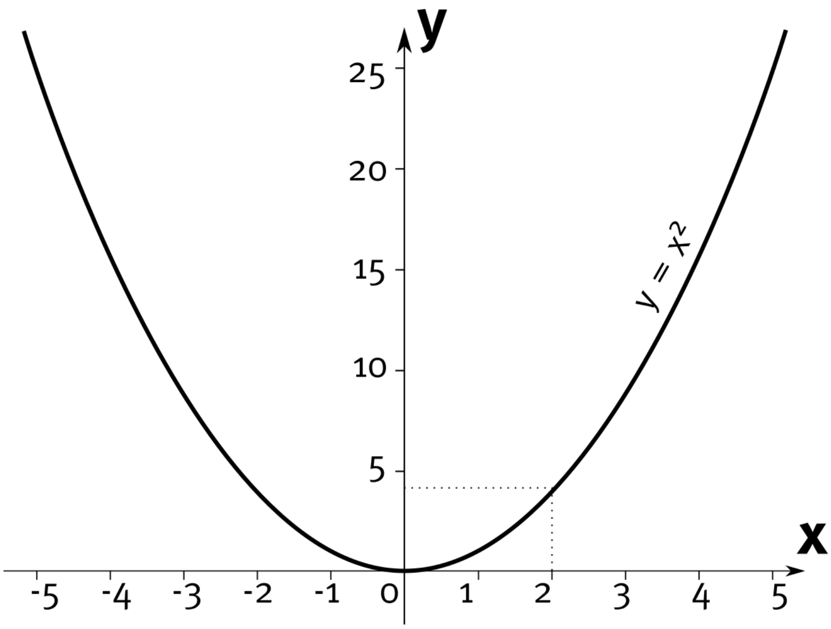
Parabolas from equations

If you learn about parabolas in high school, it'll likely be when you are studying quadratic equations - these are equations with a squared term. For example, if I wanted to draw all the points on a pair of axes x and y where y = 4, this would be a straight line, across at y = 4. If I wanted all the points where y = x this would still be a straight line, but this time a diagonal line, passing through all the points where the x value is the same as the y value - including (0,0), and (1,1) and many others.



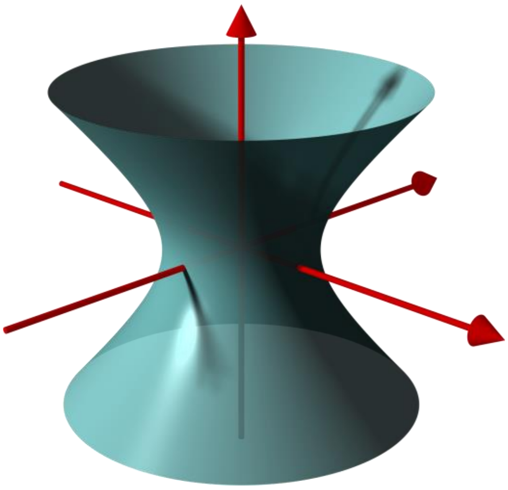


If I want to draw all the points where y = x^2, this gets a bit more difficult. We know that $2^2 = 4$ - so the line would pass through (2,4), but it's also the case that $(-2)^2$ (minus two times minus two) is equal to 4 as well - so (-2, 4) would be on our curve. In fact, the curve has two branches pointing upwards in each direction. As x gets bigger, y = x^2 gets bigger more quickly, so the lines curve upwards, in the shape of a parabola.



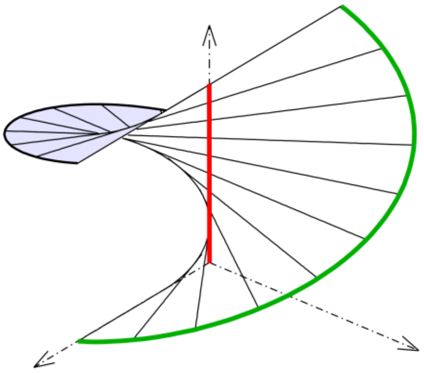
Quadratic equations (ones which include something that's been squared) crop up in all areas of mathematics - as well as modelling real-world situations, like the height of a ball that's been thrown across the room, they also occur in measurements of area, and in modelling financial systems.

Ruled surfaces



The parabola is not the only shape where a curve can be made from many straight lines. A **ruled surface** is a 3D shape whose surface is made up of entirely straight lines.

For example, if I had two circular hoops, and joined them together with a series of strings all the way round, then pulled the two hoops apart so that all the strings are held tight, they would make a shape that looks like a cylinder (although again, in order to make a true cylinder I would need infinitely many strings). If I twist, move or rotate the hoops, always keeping the strings taut between the two ends - I might have to adjust the lengths in order to do this - I can make other ruled surfaces, including the beautiful hyperboloid (pictured above).



There are many other types of ruled surface, including the cone, Möbius strip and helicoid.

Left: Helicoid, image by Ag2gaeh on Wikipedia, CC BY-SA