Big and Small Masterclass:
Extra Background Information

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Powers of ten

In this workshop, we learn about writing numbers as a power of ten. There are established names for numbers which can be written purely as a power of ten, the main ones of which are listed below.

SI units (units of measurement from the International System of units, created in 1960 as an international standard) use the same prefix for each level of number - for example, the word metre can be preceded by any of the prefixes in the table, where given, to make the name for a unit of 10 metres (decametre), 100 metres (hectometre), 1000 metres (kilometre) and so on.

Name	**Power of ten**	**Number**	**SI symbol**	**SI prefix**
One quintillionth	10^{-18}	0.000000000000000001	a	atto-
One quadrillionth	10^{-15}	0.000000000000001	f	femto-
One trillionth	10^{-12}	0.000000000001	p	pico-
One billionth	10^{-9}	0.000000001	n	nano-
One millionth	10^{-6}	0.000001	μ*	micro-
One thousandth	10^{-3}	0.001	m	milli-
One hundredth	10^{-2}	0.01	c	centi-
One tenth	10^{-1}	0.1	d	deci-
One	10^{0}	1	-	-
Ten	10^{1}	10	da (D)	deca-
Hundred	10^{2}	100	h (H)	hecto-
Thousand	10^{3}	1000	k (K)	kilo-
Ten Thousand	10^{4}	10,000	Previously a Myriad	

100 Thousand	10^5	100,000	Previously a Lakh	
Million	10^6	1,000,000	M	mega-
Ten Million	10^7	10,000,000	Previously a Crore	
Billion	10^9	1,000,000,000	G	giga-
Trillion	10^{12}	1,000,000,000,000	T	tera-
Quadrillion	10^{15}	1,000,000,000,000,000	P	peta-
Quintillion	10^{18}	1,000,000,000,000,000,000	E	exa-
Sextillion	20^{21}	1,000,000,000,000,000,000,000	Z	zetta-
Septillion	10^{24}	1,000,000,000,000,000,000,000,000	Y	yotta-
Octillion	10^{27}	1,000,000,000,000,000,000,000,000,000	B	bronta-

* This is the Greek letter mu, used to denote 'micro', as 'm' was already taken.

Under an old naming system, some of these numbers had different names - so sometimes people get them mixed up. For example, under the previous system a billion was 1 with 12 zeroes, whereas it's now got 9. Here's a guide to the old and new systems:

New name	million	billion	trillion	quadrillion	quintillion	sextillion
Old name	million	milliard	billion	billiard	trillion	trilliard
Value	10^6	10^9	10^{12}	10^{15}	10^{18}	10^{21}

None of the words jillion, zillion, squillion, gazillion, kazillion, bajillion, or bazillion (or Brazilian) are real numbers.

Scientific notation

If the number you want to write down is a very large or very small number that isn't exactly a power of ten, but you'd like to write it as precisely as you can, scientific notation is used, which is based on powers of ten. All you need to do is write the number as a number between 1 and 10, multiplied by the right power of 10. For example:

- $300 = 3 \times 10^2$
- $2554 = 2.554 \times 10^3$
- $0.00043 = 4.3 \times 10^{-4}$

- 3,628,800 (ways to order a stack of ten things) ≈ 3.63×10^6
- Avogadro's number (an important number in Chemistry) ≈ 6.022×10^{23}

In the last two cases above, you'll notice that instead of an equals symbol we've used a squiggly equals symbol. This means, 'is approximately equal to', and shows that we know this isn't the exact number. In the first case, the exact value would be 3.6288×10^6, but we have rounded the number to two decimal places. This notation is mainly used when it isn't crucial to be completely precise or exact - when you're making estimations, or dealing with numbers so big or small it would be impractical to use the exact value.

The number at the start is always between 0 and 10 (as if it is bigger than 10, you can just increase the power and divide it by 10 to get the correct value), and is sometimes called the **significand**, or the **mantissa**.

Googol and Googolplex

In 1940, American mathematician Edward Kasner published a book called Mathematics and the Imagination, in which he described a number invented by his 9-year-old nephew Milton Sirotta.

Milton had decided there should be a name for the number which is 1 with 100 zeroes after it - 10^{100} - and as this number didn't already have a name, he called it a **googol**. This is now the official name for this number - so it shows that anyone can invent something interesting in mathematics.

Milton and his uncle Edward also invented the number **googolplex**, which is 1 with a googol of zeroes after it! Both of these numbers are so big they aren't useful as a measure of any real quantity - the number of particles in the whole universe is estimated to be about 10^{80}, which is only one hundred quintillionth of a googol.

The internet company Google named themselves after a googol (spelt differently!), as they wanted to say that their search software would look through a very large number of things to find your answer. They have named their headquarters in California 'the Googleplex', as a reference to the larger number.

Estimation

We’ve been using estimation in this workshop to find approximate values, close to the actual value of a quantity that can’t be known for sure. Estimation is used in all kinds of different applications, and it’s very useful in economics and business, as it allows you to get an idea of how much something is going to cost, or how much you’ll need, before actually spending any money.

Estimates are sometimes called ‘educated guesses’ - if you were just guessing, you could say anything as your answer. Estimates are not a total guess, but you’ve used the limited information you have to ‘educate’ your guess, and make it roughly in the right area.

When you buy food in packets, the weight of food is given on the outside of the packet. But if they have to print all the packets at once, they can’t write the exact value of the weight on every single packet, and there’s no way to guarantee that there are e.g. exactly 50g of crisps in every single packet, as each crisp is a different size and weight. Instead, manufacturers use the ℮ symbol, which means ‘estimated’ - they put in crisps until they get as close as possible to the estimated weight, using careful weighing with scales. There is a legal requirement for them to be within a certain margin of the estimate, and the amount they’re allowed to be out by varies between products.

There are some circumstances where an estimate is not good enough - I’m sure if you were getting an injection of medicine from a doctor you’d prefer them to work out the exact amount you need, not just an estimate! - but there are plenty of occasions where getting a rough estimate first, then working out a more precise value later, is a very sensible technique, as it reduces wasted effort and materials if you go out and get too much or too little.

We’ve used the method of **sampling**, which means taking a small section of the thing we’re estimating, getting a roughly exact value for the sample, and then assuming that the distribution is roughly the same throughout, so we can just multiply by the proportion of the thing our sample made up.

Fermi Estimation

Sometimes people are asked to do Fermi Estimation, or answer Fermi Questions. This is a type of estimation involving answering very complex and difficult questions without having much data to start from.

Enrico Fermi was an Italian-American physicist who worked in nuclear physics, created the world’s first nuclear reactor and helped design the atom bomb. He was awarded a Nobel prize in physics in 1938, and was famous for being able to make good approximate calculations with little or no actual data.

Fermi problems help students to develop an understanding of approximation, and which pieces of information are most important and which can be guessed - including what is the most or least a value might be, if there's no way to know for sure.

A classic example of a Fermi problem is, 'How many piano tuners are there in Chicago?' To answer this you might wonder how many people live in an average-sized city like Chicago, and what number of them own pianos; how often a piano needs to be tuned (and therefore how often people actually get them tuned, given it's a costly process), how long it takes to tune a piano, and therefore how many jobs the demand for piano tuning could sustain, assuming a tuner works a standard eight-hour workday, five days a week.

Another famous example of a Fermi-problem-like estimate is the Drake equation, which asks, 'what is the number of intelligent civilizations in the galaxy'. Working out how many there might be, given the number of planets, and the probability of them sustaining life, and that life having been around long enough to develop technology and communications can often be shown to give a confusing result - the number of potential intelligent civilisations that might be able to communicate with us shouldn't necessarily be zero, all things considered - so why haven't they been in touch with us yet? This is known as the Fermi Paradox.

Other examples of estimation questions you could use

- How many bricks are in the outside of a house?
- How many coffee granules are in a jar of coffee?
- How much paper does a school use in a year?
- How much would it cost to buy pizza for everyone in this room?
- If you wanted to plant flowers all the way around the outside of this building, and they need to be spaced 5cm apart, how many flowers could you plant?
- How many students would we need to hold hands in a line that goes round the whole planet Earth?
- Estimate how many breaths you will take in a lifetime.
- Estimate the number of people who live within 1 km of your school.
- I look through a microscope and see a cell with a roughly circular cross section of 7 microns. Estimate the volume of the cell.
- The arctic tern migrates from the antarctic to the arctic. Estimate how far an arctic tern flies in its lifetime.
- Estimate the weight of dog food eaten in England each year.

- An amoeba doubles in size every 24 minutes. How long will a sample of size about 1mm by 1mm take to cover a petri dish? Do we need to worry too much about the initial size of the sample in this calculation?
- Estimate how many cells there are in your little finger
- Estimate the total weight of the pets of everyone in your class
- How much do you think your brain weighs? How much do you think this weight varies over the course of your life?
- If current trends continue what will the population of the UK be in ten years' time? Do you think it is reasonable to assume that they will continue? Why?

Accurate and Precise

You may find yourself using the words 'accurate' and 'precise' in this kind of activity - but you should be aware they mean slightly different things, and not get them mixed up!

- Accuracy is a measure of how close your answer is to the actual value - so a guess of 20 when the answer is a million would not be very accurate.
- Precise describes something which is given to a high level of detail - so, if you were trying to approximate the number 0.7734, the answer of 0.8 is reasonably accurate but not very precise; 0.33345 is more precise, but less accurate.

If I said 'I have a million kittens', or 'I have one million, one hundred and eighty-five thousand six hundred and twenty-one kittens', the second one would be much more precise (but sadly neither would be anything like accurate).

Matt Parker has made a YouTube video to explain: youtu.be/_LL0uiOgh1E