**Off the shelf Masterclass: Being Systematic**

**Session leader notes**

**Inspiration:**

The ability to work systematically, recognise patterns and extend observations to create general rules are all important skills for everyday life as well as being cornerstones of mathematical endeavour.

The aim of the Masterclass is for students to develop their skills in being systematic. They will conduct a mathematical investigation by counting squares in a chessboard, spot patterns and conjecture a rule for a chessboard of any side-length. Thinking systematically will enable them to be sure that they have counted all possible squares without missing any out or double-counting. They will extend their thinking to three dimensions by investigating cubes, starting with a 3x3 cube and developing a rule for cube of any side-length. In a longer Masterclass they will explore other two-dimensional shapes.

**General Masterclass resources needed:**

* Register of children
* Consent forms and emergency information to hand
* Stickers and markers for name badges
* Adult register
* Ri child protection policy
* Paper and pencils/whiteboards for workings
* Settling activity if not included in specific Masterclass (this one has chessboard activity)
* Drinks and biscuits

**Additional resources needed for this workshop:**

* Computer and data projector
* Coloured pencils
* Multi-link cubes
* Tracing paper
* Student worksheets

The worksheets needed for this Masterclass are downloadable as word documents from the website. They are also copied as hidden Powerpoint slides at the end of the presentation, just in case.

Student worksheets (listed in order of their use) – 1 each per student:

1. Counting squares worksheet (set up to print 2 per A4 page, will need cutting in half)
2. Results tables (double-sided)
3. Extra counting squares activity (set up to print 2 per A4 page, will need cutting in half)
4. Counting cubes 3x3x3 worksheet
5. Counting cubes 4x4x4 worksheet
6. Extra activity counting triangles

Support resources:

* Helpers notes: notes of helpers on the content of the session/activities, key questions to ask during the main activity and solutions to the problems.

**Preparation needed in advance:**

* Print worksheets and cut both counting squares worksheets in half
* Enough multi-link, either each or in pairs, to make a 4x4x4 cube (3x3x3 if short on time) – 64 or 27 cubes

**Timings:**

If you have a Masterclass which is shorter than two hours, work out which parts of the session you need to leave out. You could focus on 2D shapes, so after you finish with the chessboard problems you could explore additional shapes (slides 14, 22 & 23). You could also focus exclusively on the chessboard and the 3x3x3 cube – so leave out slide 14 and slide 17 – though you might need to spend longer looking at how to generalise for the cube. If you take out some of the session you will need to adjust the final slide (slide 19). Remember to hide the slides you are not planning to use in advance of delivering the session.

If this is not the first session in the series you will not need to include the introduction about the Ri (assuming this has been covered in a previous session).

**Ask the Ri**

Don’t forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk

**Feedback**

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

**Time plan of Masterclass:**

| **Slides & Time** | **Overview** | **Activity** |
| --- | --- | --- |
| Slide 110 minutes (10) | Introduction activityInstructions on screen.Helper and Speaker circulating and chatting with students | Working in pairs, students investigate how many squares on the chessboard. Let them explore for 5/10 minutes. They can show different squares on the chessboard by outlining them using coloured pencils.**Resources needed:**Worksheet: Counting Squares (printed 2 per page)Coloured pencilsHelper sheet |
| Slide 2 | Title slide | Introduce session to students. You may wish to give them an overview of the Masterclass. |
| Slides 3-75 minutes (15) | Introduction to the Ri | Use these slides to introduce the students to the work of the Ri and other ways they can get involved – see notes on the slides for more detail. In particular:* The Ri is a science communication charity which has been around since 1799. We’ve got a huge amount of history and lots of famous scientists lived and worked at the Ri. Most importantly, we’ve always been about communicating science to the general public – and that’s something we still do today. We do talks and activities for the public as well as with schools all across the UK.
* There are lots of family events at our building in London, including family fun days and holiday workshops just like the Masterclasses.
* The CHRISTMAS LECTURES are for young people and are on television at Christmas time, looking at a different topic every year. We’ve got an archive on our website of all of the recent series plus many of the older ones. The CHRISTMAS LECTURES are what started the Masterclass programme. See slide notes for links.
* We have a YouTube channel with lots of videos for people interested in science (and maths engineering, computer science…), especially our ExpeRimental series which is all about doing experiments at home.
* Students are part of a big family of Masterclass attendees – we have been running Masterclasses since 1981.
* Students at series running within reach of London will be invited to a Celebration Event at the Ri in June/July.
* You can become an Ri Member to get more involved with what we do (and enter the ballot to buy tickets to the CHRISTMAS LECTURES filming).
 |
| Slides 8-1015 minutes (30) | Discussion of introductory work, possible systematic approaches.How to refer to different size squares. How to record systematicallyGive out worksheet for recording results | One student can come up to the board and show some different sizes of square they have found. At this point, the students should develop a way of referring to the different sized squares. It might be worth clarifying that the sides of the squares have to be made using lines that already exist on the chessboard. **Discussion:** You can ask if any of the students can demonstrate a systematic way of investigating how many squares there are in total. What methods have the students come up with? Give them more time to explore.**Slide 10**One method that can be used is counting all the 1 by 1s, then all the 2 by 2s ….etc. With this method the next question that could be asked of the students is ‘how do you systematically count all the 2 by 2s?’ ‘If you draw the ‘first’ 2 by 2 in the top left corner, where do you draw the next one?’ Some students may draw the next square so that the edges touch and so that the two squares do not share any area. Other students will take the first square and move it along by one row so the two squares are overlapping. There is no rule saying the squares should not overlap so this is correct!**Resources needed:** Tracing paperColoured pencilsWorksheet: Results tablesPrevious worksheet: Counting squares |
| Slides 11-125 minutes (35) | Demonstration of the number of squares on an 8x8 chessboard | Now discuss how many of each square size can fit into each row/column and therefore how many can fit on the board in total. Can the students explain why the same number of squares fit along a row and down a column on the chessboard?**Slide 11 animations:** Click to show:* Results table with first row completed
* Colour flashes to show all possible 1x1 squares along the top of the chessboard; colour flashes of 1x1 squares down the left of the chessboard; number of 1x1 squares either along/up (8 squares)
* Total number of 1x1 squares
* Colour flashes of 2x2 squares along the board, then down; number of 2x2 squares either along/up (7 squares)
* Total number of 2x2 squares
* Colour flashes of 3x3 squares along; number of squares
* Colour flashes of 4x4 squares along; number of squares
* Colour flashes of 5x5 squares along; number of squares
* Number of 6x6 squares along/up
* Number of 7x7 squares along/up
* Number of 8x8 squares along/up

Can ask the students to predict the number of squares, and count along with the colour flashes (up to the 5x5 squares). Give them a minute to work out the total number of squares for the 3x3 and larger squares, and to find the total. These answers are displayed in slide 11.**Slide 12 animations:** click to show sum and final total. |
| Slide 13-1410 mins (45) | Now to generalise how many squares in an nxn chessboard?Having found the formula, it can be applied | What if the board was not 8x8? Can students work out the number of squares you would fit on a 1x1 board? A 2x2 board? A 3x3 board…can they see a pattern? Introduce ‘n’ as the side-length for a board where you don’t know how big it is – ‘n’ can be used to represent any number. Can they come up with a way to describe the smaller squares in an nxn chessboard and an overall formula?**Slide 13 animations:** click to reveal the sum and formula.Answer some questions using their new found knowledge. **Slide 14 animations:** Click to reveal the answers one-by-one.Extension for those who finish early: How many rectangles fit on the board? What do you need to think about – rectangle shape as well as size. Would you want to use any restrictions for the types of rectangles which can be used? |
| EXTRA ACTIVITY IF TIME: Slide 1510 minutes (55) | A different take on the problemIf you don’t have time for this, hide the slide when the students aren’t looking | Extension activity: A similar but different problem to tackle. Discuss where the different size squares can fit on the shape. Can you have a 5x5 square? If not, why not? Are there any places you cannot put a 4x4 square, or a 3x3 square? Note that all of the square must be contained within the shape, and as before, the edges of the squares must match the lines already there.**Resources:**Worksheet: Extra counting squares activityColoured pencils**Animations:** Click to show:* Colour flashes of all possible 1x1 squares; squares count
* Colour flashes of all possible 2x2 squares; squares count
* Colour flashes of all possible 3x3 squares; squares count
* Colour flashes of all possible 4x4 squares; squares count
* Total number of squares
 |
| 10 mins (65) | BREAK | Drinks and biscuits and comfort break |
| Slides 16-1725 minutes (90) | Now to move the systematic approach into 3D, looking at a 3x3x3 cube to start withEncourage the students to apply the same systematic approach, but now in 3d | This investigation is very similar to the chessboard investigation, but the question is ‘How many cubes in the cube?’ Start with projecting the 3 by 3 by 3 cube and asking ‘How many cubes there are in the cube’? This is the first question you are trying to investigate. Students can make the 3 by 3 by 3 cube with multi-link. Once they have made the cube, you could ask ‘why must there be more than 27 cubes?’ Can the students use multi-link to show how the 3 by 3 by 3 cube contains 2 by 2 by 2 cubes? Can they show a 2 by 2 by 2 cube by drawing on the diagram on the board? Next, ask the students ‘how many of each size are there?’ At this point the students sometimes find it useful to use colours to outline the different cubes, using the diagrams on the “counting cubes” worksheet to help them visualise. They can also use page two of the “Results tables” worksheet to record their answers.**Discuss** the results the students have found and their methods.**Slide 17 animations:** Click to show: * Each answer in the table, going across the rows.
* The sum and the total number of cubes.

**Resources:**Worksheet: Counting cubes 3x3x3Previous worksheet: Results tablesColoured pencilsMulti-link (or similar) |
| Slide 1815 minutes (105) | Look at another specific case (4x4x4) and then try to pattern spot | Can the students now do the same process for the 4 by 4 by 4 cube?**Animations:** Click to show: * Each answer in the table, going across the rows.
* The sum and the total number of cubes.

**Resources:**Worksheet: Counting cubes 4x4x4Previous worksheet: Results tablesColoured pencilsMulti-link (or similar) |
| Slides 19-205 minutes (110) | Generalising for a cubeMasterclass finish – overview, question to think about | Just as with the chessboard problem, the students may spot a pattern; they may spot that the number of cubes is a sum of cube numbers. If they spot a pattern can they explain why the pattern will definitely continue? **Slide 19 animations:** Click to show: * The table with the answers for the first few rows (1x1x1, 2x2x2, 3x3x2) & final row (nxnxn).
* The formula for the total number of cubes.

Sum-up the Masterclass and the learning points (see notes on slide).Can the students work out the relationship between the general formulae for the square and cube? (You can relate this to area and volume – can look at square side-length and area, and cube side-length and volume, if you have time and want to do this.)If we could go up to 4 dimensions, what do the students think the formula would be? Tell them that many mathematicians and physicists explore shapes in higher dimensions, even though we can only experience 3 dimensions in our universe. A 4-dimensional cube is called a “hypercube”. |
| slide 2110 minutes (120) | Thank you slide Feedback, tidy up, questions time | Ensure the students have the opportunity to ask questions – email those you cannot answer or would like additional support with to masterclasses@ri.ac.uk. Collect feedback about the class on post-it notes. We suggest asking for answers to “What was the best bit” on one colour and “What would have made this Masterclass better?” on the other. |
| Slide 22 | Possible nrich problems related to working systematicallyAdditional class activity to count triangles | Additional problems for the students to try at home:<https://nrich.maths.org/1019> One step, two step[https://nrich.maths.org/507](https://nrich.maths.org/507MSumming) Summing Consecutive Numbers<https://nrich.maths.org/1868> Shady SymmetryYou may wish to give them a print-out with these links or email the links to their teacher for them to share. |
| EXTRA ACTIVITY IF TIME: Slides 23-245-10 minutes per activity | Extend their 2D counting to equilateral triangles and right-angles triangles arranged in different patternsIf you do have time for this, move the slides to before the thank you slide  | Apply their systematic approach to squares and count triangles instead. EXTENSION ACTIVITY: What is interesting about the number of triangles as you increase the side-lengths? E.g. a triangle of side-length 2 contains 4 triangles of side-length 1. How does this relate to the area of the triangles? What is the pattern for the equilateral triangles?**Resources:**Worksheet: Extra activity counting trianglesColoured pencils |

Slides 26-33 – copies of the worksheets.