**Being Systematic Masterclass**

**Thanks for helping with this Masterclass session! Your support is much appreciated.**

The session leader should be able to tell you more about the content of the session, and exactly how they’d like you to help, but this sheet should give you some basic information you may find useful. If any of this seems obvious to you, that’s great!

In general, for Masterclass sessions:

* While the session leader is talking to the group, don’t interrupt them or distract the students unless something is wrong that needs fixing urgently. You should also watch and pay attention to what they’re saying, to set a good example.
* If things need handing out to the students, wait for the session leader to signal you to do this, as it can distract the students if you start to hand things out before they’re ready.
* If the students are given a task to work on, you should circulate the room to talk to the students. Wait until they’ve had a chance to tackle the problem before you interrupt them, and encourage anyone who looks like they haven’t started yet.
* Try not to give away the answers to the students, especially if they’re working on the problem and about to discover it for themselves - if they are really struggling, you can give them a hint or suggest where they might start looking.

**In this session:**

The aim of the Masterclass is for students to develop their skills in being systematic. They will conduct a mathematical investigation by counting squares in a chessboard, spot patterns and conjecture a rule for a chessboard of any side-length. Thinking systematically will enable them to be sure that they have counted all possible squares without missing any out or double-counting.

They will extend their thinking to three dimensions by investigating cubes, starting with a 3x3 cube and developing a rule for cube of any side-length. In a longer Masterclass they will explore other two-dimensional shapes.

**The main activities are:**

**1. Counting squares in a chessboard**

Working in pairs, students investigate how many squares on the chessboard. Let them explore – use the questions at the end of these helper notes to prompt them. They can show different squares on the chessboard by outlining them using coloured pencils, and can move on to using tracing paper. Note that the edges of the squares must match lines already on the chessboard.

One method that can be used is counting all the 1 by 1s, then all the 2 by 2s ….etc. With this method the next question that could be asked of the students is ‘how do you systematically count all the 2 by 2s?’ ‘If you draw the ‘first’ 2 by 2 in the top left corner, where do you draw the next one?’ Some students may draw the next square so that the edges touch and so that the two squares do not share any area. Other students will take the first square and move it along by one row so the two squares are overlapping. There is no rule saying the squares should not overlap so this is correct! They can use the ‘results tables’ sheet to record their answers.

The session leader will ask if anyone can demonstrate their method, and if they have a systematic way of investigating how many squares there are in total. As a class, they will discuss how many of each square size can fit into each row/column and therefore how many can fit on the board in total. Can the students explain why the same number of squares fit along a row and down a column on the chessboard? Once they have had a go there is an animation on the presentation to help them see how the squares fit onto the board.

**2. Counting squares on a chessboard – taking it further**

This may be done as a whole-class activity: What if the board was not 8x8? Can students work out the number of squares you would fit on a 1x1 board? A 2x2 board? A 3x3 board…can they see a pattern? Introduce ‘n’ as the side-length for a board where you don’t know how big it is – ‘n’ can be used to represent any number. Can they come up with a way to describe the smaller squares in an nxn chessboard and an overall formula?

Extension for those who finish early: How many rectangles fit on the board? What do you need to think about – rectangle shape as well as size. Would you want to use any restrictions for the types of rectangles which can be used?

**3. Longer Masterclasses: Counting squares extension**

This is a similar but different problem to tackle: counting the squares on a cross-shape. The students should discuss where the different size squares can fit on the shape. Can you have a 5x5 square? If not, why not? Are there any places you cannot put a 4x4 square, or a 3x3 square? Note that all of the square must be contained within the shape, and as before, the edges of the squares must match the lines already there. After they have had a go at the problem, there is another animation on the slide to help them see where the squares are.

*[This activity will only be included in longer Masterclasses, or where the whole group is working quickly]*

**4. Counting cubes in a 3x3x3 cube**

This investigation is very similar to the chessboard investigation, but the question is ‘How many cubes in the cube?’ Students can make the 3 by 3 by 3 cube with multi-link. Once they have made the cube, they may need support with breaking it down into smaller cubes. Can the students use multi-link to show how the 3 by 3 by 3 cube contains 2 by 2 by 2 cubes? The students sometimes find it useful to use colours to outline the different cubes, using the diagrams on the “counting cubes” worksheet to help them visualise. They can also use page two of the “Results tables” worksheet to record their answers.

Encourage them to use the techniques they developed in the first part of the Masterclass to consider the problem systematically. Now, however, they will need to think in three dimensions – not just looking at one face of the cube. Can the students work out how many of each size cube there are?

**5. Counting cubes in a 4x4x4 cube & generalising**

Now the students will build on what they have done for a 3x3x3 cube to a 4x4x4 cube. Can they use the patterns they spot to help them work out a general rule, just like they did for the squares? They may want to talk about square and cube numbers. The topic of area and volume might come up if they want to ask about the relationship between the how the side length and the area of the squares/volume of the cubes changes for each of the smaller squares and cubes they have been using.

**6. Longer Masterclasses: Counting triangles**

There are two different problems here where students can apply the methods of counting squares to counting triangles. These could be trickier given the shapes – so if they are using tracing paper, they may need to rotate it for the right-angled triangles. If they get through this quickly, can they come up with any rules? It’s not as simple!

*While activities 3 and 6 have been labelled here as being for longer Masterclasses, for shorter sessions or with younger students the session leader may decide to leave out activities 4 & 5 and do 3 & 6 instead. Check with them at the beginning of the Masterclass.*

**Thanks again for your help with this session! If you have any other questions, please ask the session leader.**

**Counting Squares: Useful questions**

Key assumption: The edges of squares must be made using lines that already exist on the chessboard.

* Can you draw one of each of different sized square on the diagram?
* Do you have a systematic method for counting all the squares on the chessboard? How do you know you won’t miss any out?
* How many 2 by 2 squares fit along the top row? Can you use the tracing paper to help you answer this question?
* So how many 2 by 2 squares fit on the whole chessboard?
* How many 3 by 3 squares fit on the whole chessboard?
* And continue….do you notice a pattern?
* Convince me this pattern will definitely continue.
* How many squares in total on the chessboard?

What next?

* How many squares on an n by n chessboard?
* How many rectangles on the chessboard?

**Solutions:**

1. Squares on an **8x8 chessboard** = 64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = **204 squares**

|  |  |  |
| --- | --- | --- |
| Square size | How many along/up? | How many on the whole board |
| 1x1 | 8 | 64 |
| 2x2 | 7 | 49 |
| 3x3 | 6 | 36 |
| 4x4 | 5 | 25 |
| 5x5 | 4 | 16 |
| 6x6 | 3 | 9 |
| 7x7 | 2 | 4 |
| 8x8 | 1 | 1 |

1. Number of squares in a **n x n chessboard = n2+ (n-1)2+ (n-2)2 +…..+ 22 +12**
2. Number of squares in the **cross shape** = 24+13+4+1 **= 42 squares**

(24 1x1 squares, 13 2x2 squares, 4 3x3 squares and 1 4x4 square)

1. Cubes in a **3x3x3 cube** = 33 + 23 +13 = 27 + 8 + 1 **= 36 cubes**

|  |  |  |
| --- | --- | --- |
| Cube size | How many along/up/back? | How many in the whole cube |
| 1x1x1 | 3 | 27 |
| 2x2x2 | 2 | 8 |
| 3x3x3 | 1 | 1 |

1. Cubes in a **4x4x4 cube** = 42 + 33 + 23 +13 = 64 + 27 + 8 + 1 **= 100 cubes**

|  |  |  |
| --- | --- | --- |
| Cube size | How many along/up/back? | How many in the whole cube |
| 1x1x1 | 4 | 64 |
| 2x2x2 | 3 | 27 |
| 3x3x3 | 2 | 8 |
| 4x4x4 | 1 | 1 |

1. Number of cubes in a **nxnxn cube = n3+ (n-1)3+ (n-2)3 +…..+ 23 +13**
2. Number of **equilateral triangles**: 16 + 7 + 3 + 1 = **27** (16 1x1s, 7 2x2s, 3 3x3s, 1 1x1)
3. Number of **right-angled triangles**: 16 + 16 + 8 + 4 = **44** (16 singles, 16 doubles, 8 with 4 together, 4 with 8 together).