Numbers of Nature Masterclass

Thanks for helping with this Masterclass session! Your support is much appreciated.

The session leader should be able to tell you more about the content of the session, and exactly how they'd like you to help, but this sheet should give you some basic information you may find useful. If any of this seems obvious to you, that's great!

In general, for Masterclass sessions:

- While the session leader is talking to the group, don't interrupt them or distract the students unless something is wrong that needs fixing urgently. You should also watch and pay attention to what they're saying, to set a good example.
- If things need handing out to the students, wait for the session leader to signal you to do this, as it can distract the students if you start to hand things out before they're ready.
- If the students are given a task to work on, you should circulate the room to talk to the students. Wait until they've had a chance to tackle the problem before you interrupt them, and encourage anyone who looks like they haven't started yet.
- Try not to give away the answers to the students, especially if they're working on the problem and about to discover it for themselves - if they are really struggling, you can give them a hint or suggest where they might start looking.

<u>In this session:</u>

This workshop is an exploration of mathematics in the world around us, and interesting mathematical concepts underlie many natural structures. Starting from the idea of limits of sequences, we look at the Golden Ratio, a number which has many links in nature, but is also misunderstood. We discuss why this number is interesting, and explore some examples of places it can be seen.

The main activities are:

1. **Limits of sequences**
2. **Fibonacci limits**
3. **The Golden Angle activity**
4. **Counting plants activity**
5. **Golden spiral activity**

<u>There is a great deal more background information available on a separate sheet, if you would like more detail. Please ask the session leader if you'd like to see it.</u>

Thanks again for your help with this session! If you have any other questions, please ask the session leader.

<u>In this session:</u>

Limits of sequences

We will start the session by asking the students to think about sequences of numbers - an initial set of examples will be displayed on the screen and students are asked to think about what the rule is that generates each sequence, and what comes next. If they can answer all these questions quickly, you could ask the students to think about what happens as they continue to repeat the rule to get more terms - will they continue to get bigger, or smaller, and how quickly? They can also think of their own sequences to explore.

The session leader will explain the concept of the limit of a sequence - if the numbers keep getting closer and closer to a particular number, that number is the limit of the sequence.

If a sequence doesn't keep getting closer to a particular value, that means it has not got a limit. It's not correct to say that a sequence that keeps getting bigger has a limit at infinity - some students may think this, but if it doesn't get closer to some finite value, there's no limit (and infinity is not a number!). This could be a difficult concept for some students, but in general if they can't name a particular number that the sequence is getting closer to, then it has no limit.

There will be a worksheet with several more examples of sequences, and students will be asked to write the next few terms, and then suggest what the limit of the sequence might be, if it has one (otherwise they can write 'no limit').

The next activity will be a game called **'How far do you have to go?'** This explores the formal mathematical definition of convergence, through a game in which one person has chosen a sequence with a known limit, and the other must name a small number - the person being challenged must say how far along their sequence they have to go before the distance between that term and the limit is less than the small gap they've been given.

Make sure students understand the subtle distinction between the value of their number, and the gap between it and the limit of the sequence. In many examples (e.g. 1/n) the sequence is approaching a limit of 0 from above - in which case all you need is a term in the sequence smaller than the small number given. However, if the limit is not 0, they will need to think about the difference between the terms in the sequence and the limit.

Challengers may initially name larger numbers, but as they realise this makes it easy for their friend, they may try to define smaller and smaller numbers, and depending on their understanding of how small numbers are defined they may give this as a fraction or as a decimal. You can encourage students to use a calculator, and convert any fractions to decimal so they can see the values more clearly.

You should also encourage students to answer the actual question - how far did they have to go - rather than stating a term of the sequence. They may be able to easily identify that the first term of $1/2^n$ smaller than 1/10 is 1/16, but how far along the sequence is that term?

Fibonacci limits

Next students will be introduced to, or reminded of, the Fibonacci numbers. Some may have seen them before, but they will be briefly introduced and students will be asked to find and write down the first ten terms of the sequence. Anyone who is unfamiliar may need help interpreting the instruction given on the board, which is 'Each term is the sum of the two previous terms', along with the first two terms 1, 1.

Once we have ten terms, the session leader will explain that while this sequence clearly doesn't have a limit, we're going to look at a sequence made from this one, by taking ratios of successive pairs of terms. There's a worksheet on which students must write their list of Fibonacci numbers, then calculate the ratios between each pair. They can use a calculator for this, and should give their answer as a decimal (truncating if necessary).

These terms are getting closer to a limit, although it may not be obvious how you'd find the exact value of the limit. Students should be able to see that it's not approaching the limit from strictly above or below - it moves around either side of the limit, but gets closer each time. They may be able to understand that the value it's approaching is between 1 and 2, and it's between 1.5 and 2, and it's between 1.6 and 1.66, and so on - the final two rows of the table tell them it's between 1.617 and 1.619. Referring to these numbers as ratios will help them understand the name 'the Golden ratio' for this value, as it's the ratio that is the limit of this sequence of ratios.

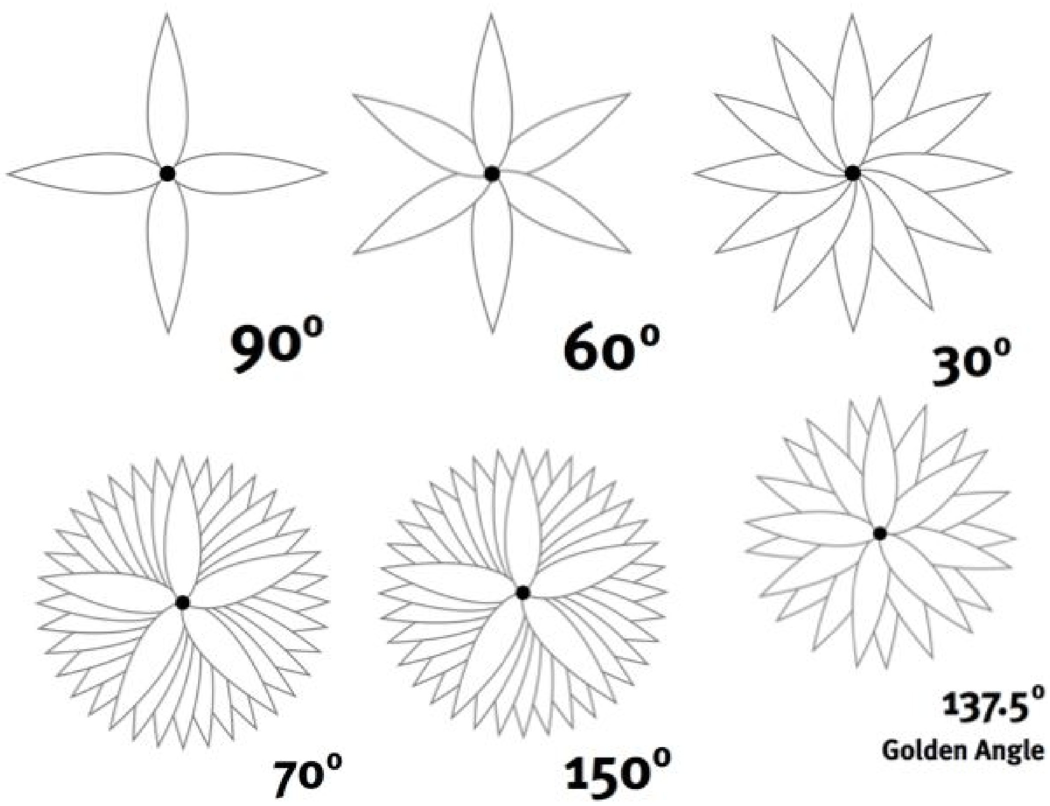
The session leader will then run through some nice properties of the Golden ratio.

The Golden Angle activity

Next we'll explore the Golden ratio as an angle. The session leader will explain how we can divide 360 degrees of a circle to get various angles, and if we divide by specifically the Golden ratio we can get a value for the Golden angle. This is strictly a value close to 222.5 degrees, but since this angle is reflex (greater than 180 degrees) and harder to work with, we can subtract it from 360 to get a more manageable angle to work with: 137.5 degrees.

Students will use a selection of different angles to create flower heads, by placing a paper cutout of the angle onto their drawing and using it to determine the position of the next petal around the circle. Once they find their next petal lines up with an existing petal (e.g. if their angle is 90 degrees and they have four petals around the circle, the fifth will coincide with an existing petal) they can stop. If any of the petal they're drawing overlap partly with an existing petal, they can draw it behind the petal that's there.

Given the angle may not be accurately cut out, and the measuring may not always be accurate, there may be slight variation in the exact positioning of the petals. Angles that are simple fractions of the whole circle should be evident, and you may be able to help students by pointing this out.



They will be given a selection of angles to try, including 90, 60, 30, 70, 150 and the Golden Angle. All the angles except the Golden Angle will give a finite number of petals (4, 6, 12, or 36) and won't look like a natural flower. The Golden Angle gives petals which are spread out, and students will find that they can draw a lot of petals without finding that two petals line up exactly.

The session leader will then be able to show students a Geogebra applet showing how a similar process creates spirals in seed heads.

Counting plants activity

The next activity will involve counting flower petals, or spirals in pine cones, and depending on what the session leader has been able to arrange, may involve the students going outside to pick their own flowers, or they can use flowers that have been brought for the purpose.

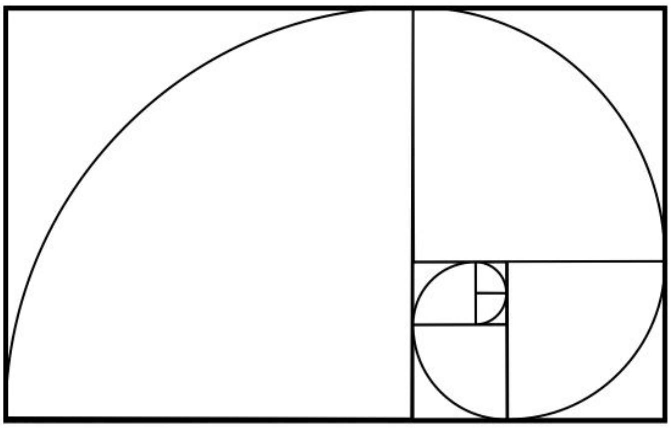
If the flowers have very small petals, it might be most practical for the students to pull off the petals and place them in piles of the same size, to make counting easier. For the pine cone spirals, they may find it helpful to draw a line along one set of knobbles with a marker pen, and count the number of parallel rows of knobbles running round the cone.

Some students will finish counting before others, and the session leader may have spare flowers/cones that can also be counted. They will invite students to come to the front and write their answers on the whiteboard/flipchart, where they will be circling any Fibonacci numbers they find (you may be able to help with this).

Golden spiral activity

To finish the session, students will be given the task of assembling a Golden Spiral, using arcs of circles with radiuses that are Fibonacci numbers. They will work in groups of four, and each group will be given a set of circles to cut into four quarters, and each student can take one quarter circle of each size (and two of the smallest one). They should cut out along the edges of the squares, not along the circle arcs, so they have a set of square pieces to assemble - watch out for anyone who cuts along the circle!

The pieces should be arranged so that the circle arcs make a continuous line (without any gaps or sharp corners), which should give them a Golden Spiral shape - this may be made in a different orientation to that pictured (rotated or reflected), but as long as it all joins up it's correct.



If they're struggling, it might help to start by placing the biggest piece into the jigsaw, and trying to join the next biggest piece so the curves match up; alternatively, they could start with the two smallest pieces to make a 2 by 1 rectangle, then place the next piece along one edge of it so the curves match and it makes another rectangle.