The Royal Institution
Science Lives Here
Masterclass network

OTS Masterclass - Numbers of Nature

Session Leader Notes

Inspiration:

Mathematics can be used to describe many aspects of the world around us, and interesting mathematical concepts underlie many natural structures. Starting from the idea of limits of sequences, we look at the Golden Ratio, a number which has many links in nature, but is also misunderstood. We discuss why this number is interesting and explore some examples of places it can be seen.

This session overlaps with 'Rabbits and Sequences', which also covers the idea of sequences and introduces the Fibonacci sequence, although both sessions could be covered by the same group, as this session focuses more on limits and covers the Golden Ratio.

Overview of Activities:

- Limits of sequences
- Fibonacci limits
- The Golden Angle activity
- Counting plants activity
- Golden spiral activity

General Masterclass resources needed:

- Register of children
- Consent forms and emergency information to hand
- Stickers and markers for name badges
- Adult register
- Ri child protection policy
- Paper and pencils/whiteboards for workings
- 2 different coloured post it note pads
- Drinks and biscuits

Specific resources needed (printable worksheets in worksheet folder):

- Worksheets:
    1. Fibonacci limits worksheet - one copy per student
    2. Limits of sequences worksheet - one copy per student
    3. Angles template sheet - one copy per student
    4. Flower Head worksheet (2 x A5) - each student will need six copies, so print three sheets per student
    5. Circle template sheets - 3 A4 sheets per group of 4 students (each student ends up with 6 quarter circles)
    6. Rectangle for golden triangle worksheet - one per student
- Calculator, one per student (doesn't need to be a scientific calculator)
- A pair of scissors per student
- Glue sticks - one per student, or one per pair
- for each student of the provided two-up sheet
- A bunch of flowers and a bag of pine cones (see below*) - either enough for one of each per student, or one of each per pair of students
- Marker pens, for drawing on the pine cones (if possible, a metallic or chalk pen will show up better); pencils may work in a pinch
- A whole pineapple and some brightly coloured electrical tape (optional)
- Whiteboard or flipchart, for collating results

* The kinds of flowers that work for this activity include lilies, irises, daisies, buttercups, wild rose, larkspur, columbine, delphiniums, ragwort, corn marigold, cineraria, aster, black-eyed susan, chicory, plantain and pytethrum. If you are near to a garden or playground where daisies/buttercups grow, you could give the students time during the break to go outside and collect some, rather than bringing your own flowers. Pine cones can often be purchased from florists or ordered online in bulk for craft purposes. If they are left to dry out, they will open up - you may need to soak them in water to get

them to close again. Then they can be dried off for use in the activity and will stay closed for a few days.

Geogebra resource: https://www.geogebra.org/m/YThycjQK#material/T8eKzDu5

Support resources:

- Helper notes: An overview of the Masterclass content and activities
- Supporting notes: Extra information and background on the content of this Masterclass
- Session script: Suggested wording for each section of the session

Things to prepare in advance

- Print worksheets and collect resources as detailed above
- Gather general Masterclass resources

Ask the Ri

Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk

Feedback

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

Time plan of Masterclass:

Slides & Time	Overview	Activity
Slide 1 5 minutes (5)	Introduction Instructions on screen. Helper and Speaker circulating and chatting with students	Settling activity - rules about sequences. Put up the intro slide with sequences on the screen, and encourage students to try to identify the rule that each sequence is generated by.
Slides 2-8 5 minutes (10)	Introduction to the Ri *[Only include these slides for the first session in the series – otherwise remember to hide the slides before you start the Masterclass]*	Use these slides to introduce the students to the work of the Ri and other ways they can get involved – see notes on the slides for more detail. In particular: • The Ri is a science communication charity which has been around since 1799. We've got a huge amount of history and lots of famous scientists lived and worked at the Ri. Most importantly, we've always been about communicating science to the general public – and that's something we still do today. We do talks and activities for the public as well as with schools all across the UK. • There are lots of family events at our building in London, including family fun days and holiday workshops just like the Masterclasses. • The CHRISTMAS LECTURES are for young people and are on television at Christmas time, looking at a different topic every year. We've got an archive on our website of all of the recent series plus many of the older ones. The CHRISTMAS LECTURES are what started the Masterclass programme. See slide notes for links. • We have a YouTube channel with lots of videos for people interested in science (and maths engineering, computer science…), especially our ExpeRimental series which is all about doing experiments at home. • Students are part of a big family of Masterclass attendees – we have been running Masterclasses since 1981. • Students at series running within reach of London will be invited to a Celebration Event at the Ri in June/July. • You can become an Ri Member to get more involved with what we do (and enter the ballot to buy tickets to the CHRISTMAS LECTURES filming).
Slides 9-29 25 minutes (35)	Discussion of introductory activity; Limits of sequences; How far do you have to go? Game	Ask the students to share their answers to the introductory sequences. What's the rule that each one is generated by? Show the slides with solutions to each sequence. Now ask the students to consider what would happen if you keep applying the rule, as many times as you want. Will the sequences keep going? Some will get smaller and smaller, and some will get bigger. There are a few different things that can happen: if something carries on getting bigger, it will just keep going forever. But if it gets closer and closer to a particular number, so that you keep getting closer the further you go, that number is called the **limit of the sequence**. Give students the worksheet on limits of sequences, and ask them to answer the questions on the sheet. If a sequence doesn't keep getting closer to a particular value, that means it has not got a limit. It's not correct to say that a sequence that keeps getting bigger has a limit at infinity - some students may think this, but if it doesn't get closer, there's no limit (and infinity is not a number!).

		Use the slides to play a few rounds of a game with the students: **How far do you have to go?** Pick one of the first four sequences from the sheet, or choose another sequence that has a limit. Ask one of the students to suggest a small number (depending on the sequence you've chosen, this might have to be less than one, and it can be expressed as a fraction or a decimal depending on the way the sequence is given). If the sequence has a limit, you should be able to say how far along the sequence you have to go to get **closer to the limit than the number they've given**. For example, if the limit is 0, you have to find a term in the sequence that's closer to 0 than the small number they've picked. You can ask the students to challenge you, and then to test each other in small groups - you and the helpers can go round and talk to each group to check their answers. For example, for the first sequence on the sheet: 1, ½, ¼, ⅛, ... The limit of the sequence is 0. If I want to get within 1/100 of the limit, I'd need to go to 1/128, which is the eighth term in the sequence. So, a reply to the challenge '1/100' would be '8'. Explain to students that this game is one way that mathematicians test whether or not a sequence has a limit - if you can always answer a challenge in this game, no matter how small a number you pick, then the sequence has a limit.
Slides 30-56 20 minutes (55)	Fibonacci limits	One special sequence that students might have seen before is called the Fibonacci sequence. Put up the slide with a rule for how to generate the Fibonacci sequence, and ask students to calculate the first 10 terms. Discuss what the limit of this sequence will be - it will just keep getting bigger, so it has no limit. Explain that a more interesting sequence can be found by dividing numbers in the Fibonacci sequence - give out the Fibonacci Limits worksheet and ask students to fill in the corresponding numbers, and divide the two values using a calculator, writing the result as a decimal in the box next to each pair. These numbers form a different sequence. The rule for it is less obvious, but the numbers seem to be getting closer and closer to a limit. In this case, the limit is called the Golden Ratio, and it's a very special number. We call it phi, which is drawn like the Greek letter phi. Ask the students to try writing the letter phi. Next, ask students to clear their calculator, press 1, then divide, and then type in the Golden Ratio from the next slide. Then ask them to add 1, and they should find they get a number that's very close to the Golden Ratio back again. This is because the Golden Ratio is the only number with the property that 1/phi = phi - 1. They can try this with other numbers - divide 1 by a number, then add 1, and see if they get back to the original number. It won't work for any other number. Show the students the next few slides, which contain other examples of ways in which phi is a very interesting number - it can be found in the shape of a five-pointed star, and in the shape shown, called a Rhombic Triacontahedron. It also has the special property that if you divide a line into two parts as shown, the ratio of the length of the long piece to the

		short piece is the same as the ratio of the long piece to the whole line - and it's the only number that has this property.
10 mins (65)	BREAK	Drinks and biscuits and comfort break
Slides 57-85 20 minutes (85)	The Golden Angle activity https://www.geogebra.org/m/YThycjQK#material/T8eKzDu5	Tell the students that next we're going to explore another aspect of the Golden Ratio - called the Golden Angle. If you divide a circle into 6 parts, you get ⅙ of a circle, and since a circle is 360 degrees, 360/6 = 60 degrees in each part. The Golden Angle is what you get if you divide a circle into phi = 1.618 parts - this isn't a whole number, but we can still divide it, so we just need to do 360/1.618 (just as we did before). Help students use a calculator to calculate the Golden Angle - it should be around 222.5 degrees, and it's a special angle in nature. Explain that we're going to investigate why some angles are more special than others, by making a flower head. We'll use an angle of 137.5 degrees as our Golden Angle, which is the opposite angle (360-222.5=137.5), and it's the same as going 222.5 degrees around the other way. Give each student a copy of the angles worksheet, and a few copies of the A5 Drawing a Flower Head worksheet. Ask them to start by using the 60 degree angle, and by placing the angle so its point is at the centre of the flower and one side lines up with the flower petal already drawn. Now they need to mark where the other side of the angle lies, take the angle away and draw a flower petal there, matching the first one. They should repeat this at least 10 times, and see what happens to the petals. If a petal overlaps where another petal already exists, they should draw it behind the one that's there, and if they like they can make it longer so it sticks further out. Once they've got the idea, they can use the other angles to try making different flower heads - which angles make a better flower? Can they see why some angles work better than others? Give them time to try different angles, and anyone who finds their favourite angle sooner can continue adding petals to see what happens. The students should find that angles which are a simple fraction of the circle - like ¼ circle (90 degrees), or ⅙ circle (60 degrees) will give boring flowers, where the petals clump in separate groups. The most interesting flowers are those where the petals are spread out nicely around the flower, and the Golden Ratio is the best angle to make this happen - you can keep drawing petals for a long time, and they won't lie on top of an existing petal. You can show the students the Sunflower Geogebra file, by visiting the link in a web browser, and explore this a little further. The seeds in a sunflower head are also arranged using the Golden Angle.
Slide 85 20 minutes (105)	Counting plants activity	Explain to the students that the angle in the sunflower is not a coincidence - it's the best angle the flower can use for placing its seeds. The Fibonacci numbers and the Golden Ratio are connected, and it turns out the Fibonacci numbers often crop up in nature as well. Hand out flowers (or ask the students to use ones picked from outside) and pine cones, and ask the students to count the petals and spirals respectively. Students with pine cones can use a marker pen to draw a line along one spiral to count from and back to, if it makes counting

		easier. There are stripes to count in both directions! The students will not all find that they get a Fibonacci number, but many of them will! Collect data from the whole group and discuss it together. Why might there be a Fibonacci number of petals? The seeds in the centre will be arranged in spirals like the sunflower, and each line of seeds will generate a petal at the end - so if there are a Fibonacci number of spirals, like on the pine cone, you'll get a Fibonacci number of petals. Why might this not happen for every single plant/flower/pine cone? If you have time and a pineapple, you could count the spirals on the pineapple together as a class. Using a strip of coloured tape to mark one spiral may help with counting.
Slides 86-90 15 minutes (120)	Golden spiral activity	Ask the students to get into groups of four, and work together on this activity. They'll need to cut out the squares, then cut each into quarters and keep one quarter each, so they each get a quarter of the circle. They can divide the cutting out between them all, until they all have a copy of each size of square (two copies of the smallest one!): 1, 1, 2, 3, 5, 8 and 13 units. They should then try to fit the squares together into a jigsaw, so that the arcs of circles all join up and create a spiral. The whole jigsaw should fit into the rectangle printed on the A4 jigsaw handout. They will find that the pieces fit exactly, because the side length of each piece is the sum of the two previous pieces together. This is because the sizes of the pieces are Fibonacci numbers. Once they're happy with the arrangement, they can glue it down. Explain to students that this spiral is called the Golden Spiral, and it's an example of a logarithmic spiral - one in which the amount that the spiral curves by changes as you go along the curve. Spirals with this property occur elsewhere in nature - in the shapes of spiral galaxies, and some seashells - but they're not all Golden Spirals!
Slides 91-94 5 minutes (125)	Feedback, tidy up, questions time Ask the Ri	Recap of session contents. Don't forget to collect any questions and feedback on post-it notes, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk
Slides 95-96	Possible NRICH problems related to this session – use as extension activities or for them to do at home	https://nrich.maths.org/8294 Making Spirals https://nrich.maths.org/4782 Rod Ratios https://youtu.be/ahXIMUkSXX0 Vi Hart - Doodling in Math Class: Spirals, Fibonacci and being a Plant