## Session Leader Notes

## Inspiration:

In this Masterclass students answer the question 'Is it worth playing the Lottery' by exploring combinations, Pascal's Triangle, probability and randomness.

## Overview of Activities:

- Students choose lottery numbers.
- Students play a probability game and discuss basic probability.
- Students discuss what we need to think about in the question 'Is it worth playing the lottery.'
- Students find combinations by hand for 'mini lottery' situations.
- Students spot patterns when these results are written in Pascal's Triangle format.
- Use Pascal's Triangle to find how many ways of choosing 6 numbers from 59 there are.
- Think about what the probability of winning the jackpot means in 'real life' terms.
- (Additional material: play a game to understand expectation, and show students what the expected value of a lottery ticket is).
- Students choose their favourite of the 4 groups of lottery numbers. Discuss equal likelihood of each group being the winning numbers, and which numbers are popular choices amongst players using news story and graphs to discuss this.


## General Masterclass resources needed:

- Register of children
- Consent forms and emergency information to hand
- Stickers and markers for name badges
- Adult register
- Ri child protection policy
- Paper and pencils/whiteboards for workings
- 2 different coloured post it note pads
- Settling activity if not included in specific Masterclass
- Drinks and biscuits.

Specific resources needed (available on slides or separately in worksheet folder):

- Set of PowerPoint slides for session leader.
- Flip chart/white board or interactive white board
- (Optional) "Iottery card" for start
- A coin to flip
- Print out of Pascal's Triangle worksheet, one per student
- Print out of More Combinations worksheet, one per student
- Students will need calculators
- You may wish to find an interactive Pascal's Triangle online
- Extension material of 'Expected Value' requires sweets
- Printout of the take home activities.


## Support resources:

- Helper notes: An overview of the Masterclass content and activities.

Things to prepare in advance

- Print worksheets and resources as detailed above
- Gather general Masterclass resources.


## Ask the Ri

Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk

## Feedback

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

In the session guide below important questioning and dialogue is in italics and speech marks. Outlines of student activity and other teacher activity is then in normal type.

## Time plan of Masterclass:

Times in brackets indicate possible overall length if all material included, ** activities/sections can be removed without overall thread of Masterclass being lost.

| Slides \& Time | Overview | Activity |
| :---: | :---: | :---: |
| Slides 1 (As students arrive) | Entry activity | Choose lottery numbers |
| Slides 2-8 <br> 5 <br> minutes | Introduction to the Ri <br> [Only include these slides for the first session in the series - otherwise remember to hide the slides before you start the Masterclass] | Use these slides to introduce the students to the work of the Ri and other ways they can get involved - see notes on the slides for more detail. In particular: <br> - The Ri is a science communication charity which has been around since 1799. We've got a huge amount of history and lots of famous scientists lived and worked at the Ri. Most importantly, we've always been about communicating science to the general public - and that's something we still do today. We do talks and activities for the public as well as with schools all across the UK. <br> - There are lots of family events at our building in London, including family fun days and holiday workshops just like the Masterclasses. <br> - The CHRISTMAS LECTURES are for young people and are on television at Christmas time, looking at a different topic every year. We've got an archive on our website of all of the recent series plus many of the older ones. The CHRISTMAS LECTURES are what started the Masterclass programme. See slide notes for links. <br> - We have a YouTube channel with lots of videos for people interested in science (and maths engineering, computer science...), especially our ExpeRimental series which is all about doing experiments at home. <br> - Students are part of a big family of Masterclass attendees - we have been running Masterclasses since 1981. <br> - Students at series running within reach of London will be invited to a Celebration Event at the Ri in June/July. <br> - You can become an Ri Member to get more involved with what we do (and enter the ballot to buy tickets to the CHRISTMAS LECTURES filming). |
| $\begin{aligned} & \text { Slide 9-14 } \\ & 15-20 \\ & \text { minutes } \\ & (20-25) \end{aligned}$ | Introduction (If struggling for time remove first two bullet points and start from 'Introduction to objective to Masterclass' | - **Heads and Tails Game <br> - **Discuss basic probability with reference to the game <br> - Introduction to objective of Masterclass |
| Slide <br> 15-25 <br> 20 mins <br> (40-45) | Combinations | - Introduction to combinations <br> - More combinations (table) |


|  <br> Time | Overview | Activity |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { Slide } 26-34 \\ & 25 \text { mins } \\ & (1 \mathrm{hr} \mathrm{5-10}) \\ & \hline \end{aligned}$ | Pascal's Triangle | - Fill in Pascal's Triangle <br> - Patterns in Pascal's Triangle <br> - Pascal's Triangle and lottery probability |
| Slide 35-38 5/10 mins (1hr 10-20) | So, is it worth it? | - Express probability as a decimal and find out how many years and lifetimes it takes us to expect to win the lottery once. |
| Slide 39-54 25-30 mins <br> (1hr 35 - 50) | **Expected value (extension material for a longer session) | - Expectation games <br> - Expected value of a lottery ticket <br> - Expected value of ticket news story |
| ```Slide 55-60 20 mins (1hr 55-2hr 10)``` | Popular Numbers | - Play the lottery <br> - *Which numbers would they choose <br> - *Charts and news stories |
| Slides 61-62 | Suggestions of tasks to extend learning | Suggestions of tasks to extend learning |
| Slide 63 | Feedback, tidy up, questions time <br> Ask the Ri | Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk |

## Session Guide

Suggested script in italics.
In a section that involves a worksheet, it will say 'Worksheet needed' before the slide numbers.

## Introduction

Optional "Lottery card" to fill in could be printed. (Slide 1) Entry Activity:
'Write down the 6 lottery numbers you would choose if you were playing the lottery, and save them for later on.'

## (Slide 9 - title slide) Introduction:

'This session explores what maths says about the National Lottery and whether it is worth playing. This involves thinking about probability. We are going to start with a warm up game to get us thinking about probability.'

## Play Heads and Tails Game:

Choose a student to flip a coin.
'Everyone else stand up. The student is going to flip the coin, but before that happens you are all going to make a prediction about what will happen. You will put your hands on your head if you predict heads, and hands on your back (your 'tails'), if you predict tails. The coin will then be flipped, the result revealed, and whoever predicted incorrectly will sit down. The game will continue like this with you all predicting the flip results, and then sitting down if you are incorrect, until there is only one person left who is the winner.'
Play the game! Get a winner.

## Discuss basic probability with reference to the game:

'I want to work out how lucky a person would have to be to predict the result of (e.g) five coin flips. To do this we'll need to consider how many different possible outcomes (sequences of $H$ and $T$ ) we could get when we flip a coin eg five times.'
'We could break this question down by first asking if I just flipped the coin once how many possible outcomes there would be. What different things could happen?'
(Slide 10): Students will be confident with Heads and Tails.
'Because there are only two equally likely options for what could happen, that is why the probability of getting a Tails is one out of two.'
(Slide 11/12): 'Can you list all the possible outcome for two coin flips?'
When they give their answers emphasise that the order matters here. Then reveal answers slide.
`Can you list all the possible outcome for three coin flips? How will you know when you have found them all? Make sure you have a system when you are writing them out.'
(Slide 13) Show the answers.
'Can anyone tell us, without writing them out, how many outcomes there will be for four coin flips and explain why?'

Students will probably spot that the total is doubling each time, but ask if a student can explain why the doubling happens.
Explanations include: 'You can have any of the eight options for the first three coin flips, but then there are two options for what could happen in the fourth coin flip. This means there are $8 \times 2=16$ options in total for four coin flips. For the same reason, there would then be 32 options for five coin flips.'

In this discussion, you can then convince the students how many possible outcomes there would be for the number of coin flips (eg five) that they did in the game.
'So, the probability that a particular person would correctly predict that many coin flips in a row is one out of eg 32, because there were 32 possible outcomes.'
Write this fraction on the flip chart/white board again to reinforce that form. Remind them the line means divide so we can do numerator divided by denominator to turn the fraction into a decimal.
'Is this closer to impossible or certain?'
Ask a student to explain their answer, to remind others that the probability line starts at 0 and ends at 1.

## (Slide 14) Introduction to the objective of the Masterclass:

'In this Masterclass we will be answering the big question: if we are looking to make money, is it worth playing the lottery? Discuss with the person next to you what we need to think about in order to answer that question.'
It's probably worth writing down all the student suggestions, making it clear you are not confirming whether their suggestion is a factor in whether it's worth playing the lottery or not. Then refer back to the comments throughout the class so the students can find out if their point was important.
Factors to consider include: 'how many ways of choosing six numbers from 59', 'the probability of winning', 'what you can win' and 'how much it costs to play.'
Students often think 'the number of people who play' affects your chances of winning the jackpot. Its good to call back to this after you have discovered that your chance of winning the jackpot is independent of the number of people who play. However, do point out later that how much you might stand to win can be affected by how many people play, so it is not irrelevant. This can be discussed in the popular numbers and (expected value - extension material) sections.
Now, draw on a response suggesting you need to work out 'how many ways of choosing six numbers from 59.'
'We are going to work out how many ways there are of choosing six groups of numbers from 59, to work out your chances of choosing the correct six and then winning the jackpot.'

## Combinations

## (Slide 16) Introduction to combinations:

'Often when mathematicians have a difficult problem like 'how many ways of choosing six numbers from 59', we simplify the question and build up a solution from there. We're going to do that, and start by considering a mini lottery.'

Students have a go at the questions on their white boards/scrap paper. (The mini lottery has numbers $1-5$ in it, and 2 are chosen for the game)
'In the lottery the numbers can only be chosen once each, and the order they come out does not matter.'
After they have had a go on their own/pairs, remind students 'we don't count both 1,2 and 2,1 because order doesn't matter. And we don't count pairs of the same number such as $\mathbf{( 1 , 1 )}$ because numbers can only be chosen once.'
(Slide 17) Show your list of pairs.
'See how I didn't just think of the pairs randomly, but had a system: I found all those involving one, then all those involving two, etc. This way I know I've got them all. There are 10 pairs in total. Mathematicians summarise this result by writing: $\mathbf{5 C 2} \mathbf{C 1 0}$ where this means there are 10 ways of choosing two numbers from five. We call it 5 choose 2. The C represents choose.'
'At this point, somebody usually thinks its 10 because $5 \times 2=10$. But that's just a coincidence. Its equal to 10 because we found 10 ways to choose the pairs of numbers.'
Click so the triangle shape appears.
'We can arrange the outcomes into a triangle shape - and this means the number of combinations is a triangular number. $10=4+3+2+1$, and that's how you make triangular numbers, by adding consecutive numbers.'
'In fact - the number of ways we can choose two from a larger group will always be a triangular number.'
'Now, can anyone use the triangle shape to tell us how many ways we can choose two numbers from a group of four?'

## Click to show 4 C $2=$

Once a student has explained then reveal the smaller triangle for $4 \mathbf{C} 2$ and the answer, 6.
A suitable explanation would be: 'now there are only four to choose from we can remove the end outcome (the one involving five) from each row. So, there are $3+2+1=6$ combinations - still a triangular number.'
(Slide 18) Show the combinations for the extension question.
'There are 10 ways and therefore $5 \mathbf{C} \mathbf{3}=\mathbf{1 0}$.'
'What is the chance that the winning three numbers is $1,2,4$ ?' Students should say 1 in 10 .
Now ask 'what is the chance that the winning 3 number is $1,2,3$ ?' Students will say 1 in 10 .
Repeat for another choice.
'So, they are all equally likely? And it doesn't matter what numbers they are - for example, consecutive numbers are just as likely to come up?' They should conclude yes!
This conversation is referred to later on when students are less convinced that all groups of lottery numbers are equally likely to come up.

## Resource needed. (Slide 19-25) More combinations:

'It's going to take us a long time to do the same thing for the bigger numbers in the real lottery, so instead we're going to do it again for smaller numbers and see if we spot any pattern in our results.'
Explain how the table works, pointing out the results they have already seen.
Then click the black shading in.
'What does this black represent?' Students should tell you these boxes are impossible to fill in. Then click the blue shading in.
'And the blue shading denotes the column that everyone argues about!'

Set the students off to fill in as much as they can, giving them the worksheet and encourage them to write out the combinations on scrap paper if they need. Ask the students who have finished to spot patterns and see if they can explain them.
Now reveal answers, spending less time on the easier ones.
For the blue column, ask for a show of hands for 1,0 or something different. Lots of students think it will be 0 , rather than 1 !
One way to explain why 'something choose 0 ' is 1 is: 'if you have three students and you have to choose zero of them, there is one way of doing this: walk away! Don't choose any of them.'

## Pascal's Triangle

Resource needed. (Slide 27) Tell the students 'If we shift the grid a bit and put it in this form it might be easier to spot some patterns.' Reveal Pascal's Triangle.
'In this form the grid has another name - Pascal's Triangle.'
Fill in Pascal's Triangle: Give students the Pascal's Triangle sheet. You could mention that Pascal's Triangle was in fact discovered in China by Yang Hui 500 years before Pascal wrote about it.
'Can you follow any patterns you see to fill in the remaining rows.'
If students finish early, you could ask them to colour in all the odd numbers and see if they spot any patterns (they get a fractal pattern).
(Slide 28) Patterns in Pascal's Triangle: The next slide allows you to reveal different cells of the next row of Pascal's triangle.
'Most of you spotted that the end cells are 1, and 6 is in the cells just inside of those so that the run of consecutive numbers continues.' Click these in.
Then reveal the yellow shading.
'What is special about these numbers? How can you continue the pattern?'
Answer: 'The yellow cells are all triangular numbers which is why 15 follows this.' Click in 15.
Then reveal the green shading.
'What connects the three green cells?'
Answer: 'They are connected because the two cells above add together to give the cell below - this works for every set of 3 cells arranged like this in Pascal's Triangle.'
Then reveal the next three green squares and show that 10 plus 10 gives 20 as the final missing cell in that row.

(Slide 29) More of Pascal's triangle is revealed so students can check their answers. There are other patterns that can be pointed out:
Click so the first two consecutive triangular numbers are highlighted in blue.
'What do these two numbers add up to?'
Do the same for the next pair, and the next.

Ask the students 'what is special about the numbers you all just said.'
Answer: 'They are square numbers! Every pair of consecutive of triangular numbers adds up to a square number.'

Then click in the first set of green and yellow dots.
'one and three are triangular numbers because you can arrange each of those number of dots into a triangle. And you can see here that if we put the two 'triangle' shapes together, we get a square.'
Then click in the next layer of green and yellows dots.
`And it works for the next pair of triangular numbers. Put the two triangles together and you get a square. That's why pairs of consecutive triangular numbers add up to a square number.'
(Slide 30)
'There are other fun patterns in Pascal's Triangle.'
'What is 11 to the power of 2?'
Then click to bring in the text box with this result in it, it appears next to row 2 of Pascal's Triangle, where the digits 121 appear.
'What about 11 to the power of 3.'
Do the same thing again, once they have answered click in the text box - it will come in next to row 3 to show the students where it appears in Pascal's Triangle.
Do the same thing again for 11 to the power of 1 and 11 to the power of 0 . The students are less likely to know these ones.
'So row 0 is equal to 11 to the power of 0 , but actually its equal to anything to the power of 0 , including 2 to the power of 0 .'

Then click in the 2 the power of 0 text box.
'Add up the digits in the next row.'
Click to reveal the answer. The click again to reveal that this is the same as 2 to the power of 1.
Then repeat this for the next two rows!
(Slide 31) Explain on this slide how you can use Pascal's triangle to look up answers to different combinations questions.
Point at a number like the 6 (from 4C2) in Pascal's triangle.
'Where is this 6 in the grid?' Students say where they see 6 in the grid.
Then 'so this 6 is the answer to 4 choose 2. And it's in row 4. Its row 4 because we start at row 0 . So, if I wanted to find 4 choose something and didn't have the grid, I can go to row 4 of Pascal's triangle. The 6 is 4 choose 2 but it's the third cell in. That seems odd. Can anyone explain why it's the third cell in, rather than the second cell in?'

Answer: 'We always count from 0 , so the first cell is 4 choose 0 , the second is 4 choose 1 and the third is 4 choose 2.'

Do another example.
(Slide 32) Pascal's Triangle and lottery probability: Challenge the students to use Pascal's triangle to find the probability of winning the mini lottery.

Once students have read the instructions and had a go at finding 7 choose 2, then lead them through it again, counting down to row 7 from row 0 and in to cell 2 from cell 0 .
'There are 21 ways of choosing two numbers from seven, therefore the chance of winning the jackpot in this mini lottery is 1 over 21.'
'For part three, we can say we would expect one out of every 21 people on average to win, and there are 40 (for example) people in the room, so that's probably about two of you that we would expect to win.'
(Slide 33/34)
'So, what could we do to find the probability of winning the jackpot on the real national lottery?' 'We need to use Pascal's Triangle. Which row do we need to go to?'
'First, have a guess at how many ways you think we can choose six from 59.' Hopefully students' answers will be low, so it's a bit of a surprise when the true number is revealed!
'Now let's reveal the answer.'
Use the Interactive Pascal's Triangle, and make it show row 59 on the screen, then count along to cell six with the students.

Alternatively, slide 34 shows the value of the two cells that sit above the $\mathbf{5 9} \mathbf{C 6}$ cell so the students can add them up to find its value.

## So is it worth it?

(Slide 36)
'So, remember we are trying to conclude whether it is worth playing the lottery. And to answer that we needed to look at the probability of winning the jackpot. 45,057,474 is the number of ways of choosing six numbers from 59, but what does that mean for the probability of winning the jackpot?

Answer: 'The probability is one out of that number.' Click in the next two text boxes.
'And we know that fraction line means divide, so we're doing one divided by 45,057,474.' Click in the decimal.
'This is very close to zero/impossible!'
(Slide 37)
'This probability is very small, but it's still hard to understand whether that means over my lifetime it's worth playing or not. So, I'm going to ask you some questions that make you think about what this means for real life.'
'Firstly, how many times would you need to play to expect to win once?'
If students struggle with this concept, say something like 'well, how many times would you need to roll a dice to expect to get say a three once?'
Answer: 'Six times, because the probability is one out of six, and similarly you'd need to play the lottery 45,057,474 times in order to expect to win once, because the probability of winning is one out of $45,057,474$. It might be you still don't win in that time of course, or it could be you win first time, but on average we expect people to need to play 45,057,474 times to expect to win once.'
'Using this fact answer the questions below. You can use calculators, and you'll need to make some estimates/assumptions.'
(Slide 38)
Talk through the answers.
'How many weeks are there in the year?'
'So we're going to divide the number by 52 to find out how many years we'd need to play for.' Click in first answer.
'What did you divide by next?'
Students often suggest lots of different life expectancies and assume that we divide by the life expectancy.
`I assumed life expectancy was 90 years, but despite that I've divided by 74. Can anyone suggest why?'

Answer: 'Because you have to be 16 to buy a lottery ticket. So, you only have 74 lottery playing years in your life!' Click in the next calculation, and then click in the answer for the final question.

After revealing the number of lifetimes it would take to expect to win the jackpot once, ask the students:
'So, is it worth it?'

## Extension material: Expected value of a lottery ticket

(Slides 39 - 42) This section could be involved in a setting with primary students who are particularly advanced, engaged and have a really quite long session.
'The question we are trying to answer is 'Is it worth playing the lottery?' and we've considered your chances of winning the jackpot. Is there anything else that can happen if you play the lottery? Are there any other outcomes that we should consider?'
'We haven't taken into account the fact that there are other prizes on offer for matching fewer numbers. There is a mathematical measure we can use to work out if playing a game with multiple outcomes is worth it. We're going to introduce this by playing some games.'

Expectation Games: Choose three students to come out to the front.
'I'm going to give you each three sweets. This is your currency to play a game with - you can't eat them yet!'
'Student 1 you have the option of playing Game 1 - read the instructions. So, you can choose to spend your sweets and play, or keep your sweets and not play.'

Before the student decides, ask the audience: 'do you think it is worth it? Should they play?'
Student then plays/doesn't play the game. Remember if they play to replace the counter that is chosen from the bag.
Do the same thing then for the subsequent two games/students. The first game will have lots of hands up - most students will think it's worth playing. The second and third will have fewer hands up.
'Lots of you put your hands up for the first game, but fewer of you put your hands up for game two and three. Why is that?'
Students will probably tell you they didn't raise their hands for Game 2 because the prize was lower, and for Game 3 because the probability was lower. This means you can establish that the prize and probability are important.
'So, you're telling me that the size of the prize and the probability of winning are both important factors. There is a mathematical measure of if a game is worth it that considers both the probability of winning and the prize. This measure is called expected value. Mathematicians would calculate the expected value of the game, and see whether that value is greater than what you pay to play. Which I think is what lots of you did, maybe without realising it. Here's an example.'

## (Slide 43 - 45)

In the next section, click things in as you say them. 'So the formula for expected value is probability of winning multiplied by the prize. In game 1 here, the prize was 10 sweets and the probability of winning was a half. If we multiply these together we get five. Five is the expected number of sweets you get for playing the game. Since you paid three sweets to play and five is higher than three, that's why you thought it was worth playing.'

Then do the same for the next two slides which are game 2 and 3, revealing that the expected value is lower than the number of sweets it cost the students to play. That's why not many people thought it was worth playing these games.
You'll notice the calculation for expectation here involves just multiplying the prize by the probability of winning, and does not consider the outcome of losing.

## (Slide 46-47)

'In actual fact, there is still an outcome that we have forgotten about in these games. That is the outcome of losing. The formula we used should actually have been this.'

Click in formula that uses subscripts.
'This is the same formula but with algebra to stand for the words. For example, Pw stands for the probability of a win.'
'Let's fill that in for game 1. The first bit is the same as before. Then the probability of losing is a half, but the prize you get if you lose is zero. And multiplying zero by a half, gives zero. So that bit disappears anyway! And that's why it didn't matter that we ignored the outcome of losing earlier on.'
'But the real formula does involve all possible outcomes, and in some games there are more outcomes than just winning and losing. For example, in the lottery you could match all six numbers, or five numbers...etc.'

## (Slide 48-53) Expected value of a lottery ticket:

'We are going to use this method to find the expected value of a lottery ticket. If the value is greater than the $£ 2$ it costs to buy a ticket, then it's worth playing.'

Explain how the table works. Then reveal the probabilities for each of the outcomes.
Students could then do the multiplication and addition themselves, or just the addition, or neither.
'So, the expected value of the lottery ticket is 36p!'
Click in the next slide (slide 53) and conclude this section by reading this slide and asking the students again if it is worth playing the lottery.

## (Slide 54) Expected value of ticket news story:

'This news story from January 2016 shows that it is possible for the expected value to change. In this story the expected value was greater than $£ 2$. What might have caused that?'
'The expected value is dependent on two things, remember: the value of the prize money and the probability of each outcome. We used the average prize per person for our calculation but the prizes are not always equal to these averages. They are dependent on how big the pot of money is, which depends on roll overs and how many people play. In this particular week, there had been lots of roll overs, so the prize money would have been higher than the average. The probability would not have changed - that stays fixed.'

## Let's Play the Lottery!

(Slide 55-57) Play the lottery:
'Let's play the lottery - get out your lottery numbers!'
You can play the short video which reveals a set of winning numbers. You need to press play on the bar at the bottom of the video. If you are struggling to make it work, then just reveal the next slide which shows the winning lottery numbers.

## (Slide 58) Which numbers would they choose:

'Nobody won! But imagine if you had to play the lottery for real. And imagine you had to choose one of these groups of numbers - which one would you choose? Make a decision with the people on your table: which one you would go for and why.'

Then take a show of hands for each group of numbers. Often lots of students choose group A. Ask a student who chose A why they chose A. Students often talk about group A being more likely to come up because they are more 'spread out' or more 'random.'
'Do you remember during the mini lottery at the beginning of the class I asked you whether 1,2,3 was equally likely to come up as $1,2,4$ ? You all said the outcomes were equally likely. It didn't matter whether the numbers were consecutive or not. They were all one of the ten ways of choosing three numbers from five. It's exactly the same thing here, each group of six numbers is equally likely to come up. They are all one out of the 45 million ways of choosing 6 numbers from 59.'
'So in terms of your chances of winning the jackpot or other prizes, the numbers you choose does not make a difference.'
'Each of the groups of numbers are equally likely to come up, but is there any other reason you might prefer one group over another?'
'If you do win, then the more people who win with you, the more people you have to share the prize with. So it could be argued that its worth choosing numbers that are unpopular with other people.'
Students will then often argue that $1,2,3,4,5,6$ is unpopular (because people think they are less likely to come up, like they did!) and therefore a good group to choose.
'In fact, 1,2,3,4,5,6 is one of most popular groups precisely for this reason - lots of other people have thought of this and think they are being clever!'
'So this leaves group C and group D -what is different between these two groups? Why might you be more likely to choose one rather than the other?'
'It could be argued that choosing group D, with higher numbers, is a good idea, because lower numbers are in general more popular than higher numbers. In particular, people often use family members' birthdays for their lottery numbers and these are all lower than 31.'
(Slide 59) Charts and news story: Show the students this chart.
'This is the chart of how often each lottery number has been chosen by Masterclass students doing this Masterclass across in the country a few years ago. What do you notice? Which numbers are more popular?'

In addition to lower numbers being generally more popular than larger numbers, there are some other interesting things students can notice: there appears to be an 'I haven't had a large one - add in 59 for good measure' thinking going on - 59 is unusually popular, 10 is particularly unpopular amongst the low numbers and seven is the most popular number overall by a considerable margin.
(Slide 60)
'Seven is popular not just with Masterclass students but with the population as a whole.'
'What do you notice about the numbers that came up on 23 March 2016?'
'They were almost all multiples of seven! And because people like seven, lots of people matched some of the numbers. In particular, so many people matched five numbers that the winners only got $£ 15$ each. Considering they managed to do something that had a 0.0000069244 chance of happening, a reward of only $£ 15$ is not a great result.'
'What did you choose? How many of you used birthdays?'
'Did any of you choose consecutive numbers?'
'And crucially, how many people chose seven?'
Finish by concluding, that it's not worth playing the lottery if you're looking for a money-making method.

