

Primary OTS Masterclass: The Mathematics of Bees

Thank you very much for leading this Masterclass. We hope that you enjoy working with this material as much as we enjoyed putting the activities together. We do appreciate all the effort that our volunteers put into bringing inspirational Mathematics Masterclasses to students around the country.

Inspiration for this topic:

Honeybees build their honeycomb from thousands of hexagonal cells, creating a structure that is both strong and efficient. This workshop explores the mathematics behind this remarkable natural design.

In this workshop, students will explore tessellations and compare how different shapes share edges which increase their storage and material efficiency. By testing triangles, squares and hexagons, they will discover why hexagons allow bees to build efficient honeycomb using the least amount of wax.

This workshop has been developed in collaboration with the [Natural Curriculum](#), and Educational Recording Agency (ERA). With an [ERA Licence](#), your school can access all of the Natural Curriculum resources free of charge. The vast majority of educational institutions in the UK have an ERA Licence, but if you need to check, click [here](#).

Overview of Activities:

1. Starter Activity – Worksheet 1: Shapes in Nature
2. Shapes in Nature video
3. Discussion on bees and hexagons
4. Honey hunters in Nepal video
5. Tessellation investigation activities
6. Worksheet 2: How much can each shape hold?
7. Behind the beehive video
8. Bubbles to Hexagons demo
9. Playdoh hexagons activity
10. Extensions and Feedback – Worksheet 3: Hidato beehives and Worksheet 4: How much wax?

General Masterclass resources needed:

- Children and adult registers, consent forms and emergency information to hand
- Stickers and markers for name badges
- Paper and pencils
- 2 different coloured post-it notepads (for feedback at the end)
- Drinks and biscuits

Specific resources needed (tailor to the number of students attending):

- 1 copy of Worksheet 1 and 2 per student
- Rulers, 1 per student
- Small counters (or jellybeans) for Worksheet 2
- Washing up liquid and water mix
- Paper plate (1 for demo, or 1 per student)
- Straws (1 for demo, or 1 per student)
- Playdoh
- Video clips available at www.naturalcurriculum.co.uk/ri-and-nc-masterclass/ri-mathematics-of-bees/ and an [ERA licence](#).

Optional:

- Laptops, computers, or tablets with internet access, 1 per student ideally
- Polydron kits, one per group of four students
- 1 copy of Worksheet 3 and 4 per student – you may wish to laminate these to use them again.
- Calculators (for worksheet 4)

Support resources:

- PowerPoint slides
- Session Script

Things to prepare in advance

- Gather the complete list of resources as detailed above
- Print worksheets, additional information, and any other resources as needed

Ask the Ri

Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: masterclasses@ri.ac.uk

Feedback

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

Time plan of Masterclass:

Slides & Time	Overview	Activity (see script for further details)
Slides 1-11 10 minutes (10)	The Mathematics of Bees Introduction Worksheet 1 – Shapes in Nature	Hand out Worksheet 1: Shapes in nature as a starter activity for students. Ask the students to work out what each of the zoomed in images of nature show. Welcome students to today's Masterclass. Explain that they will be exploring the mathematics behind beehives in today's session. Play video clip 1: Shapes in Nature, available at www.naturalcurriculum.co.uk/ri-and-nc-masterclass/ri-mathematics-of-bees/ . You will need to sign in with an ERA licence to watch the clip. You may also find the clip at https://www.bbc.co.uk/iplayer/episode/b07k7m4z/forces-of-nature-with-brian-cox-1-the-universe-in-a-snowflake (from 14:53 to 16:03). Go through the answers to the starter worksheet. Click to move onto the next slides to show the answer to each image, from 1 – 9.
Slides 12-13 10 minutes (20)	Bees – what do we know? Discussion activity Bees and hexagons	Explain that whilst lots of shapes appear all over the natural world, we are going to focus today on the shapes that help bees. Ask the students what they already know about bees. After some discussion, click to go through some key facts about bees. Explain that some bees do choose to live solitary lives and make nests in soil, wood, or stems, but the social honeybees and a few other bees (stingless bees for example) create hexagonal honeycomb hives. Ask the students what they know about a hexagon – it has six sides. Explain that a perfect, or regular hexagon, has six sides of equal length. These wax cells are used to store honey and raise larvae, young bees. Go to slide 13 and pose the question of why bees from different areas of the world make the same hexagonal pattern, but do not give the answer away yet. Remind the students that a hexagon is a 6-sided 2D shape. Ask the students what we call a 3D version, a shape with a hexagon base – a hexagonal prism.

Slides & Time	Overview	Activity (see script for further details)
Slides 14 – 16 10 minutes (30)	<p>Video Clip 2: Honey hunters in Nepal</p> <p>What do bees need discussion?</p> <p>Efficient shapes</p>	<p>Play the Honey hunters in Nepal video clip, which can also be found here: https://www.bbc.co.uk/programmes/p03zn0yn.</p> <p>After the clip, discuss what the students could see. Highlight that in the clip, Brian Cox explains that for every gram of wax a bee produces and uses to build their hive, they will have to use more than six grams of honey. This is why bees need to build efficient hives, using as little wax as possible. Explain that today they are going to see investigate how bees do this!</p> <p>Move to slide 15 and ask the student to discuss with the people around them, if they were designing a home to house thousands and bees and lots of honey, what would they need to consider when designing the size, layout and shape of the wax cells. After discussion, take in responses, and then click to go through some suggestions on the slide.</p> <p>Move to slide 16 to explain that if wax cells have gaps between them are inefficient. Introduce the term tessellate, and explain that it means when a shape can be repeated to form a pattern with no overlaps of gaps between each individual shape. Click to show the example hexagons tessellating, and the circles with gaps between them, and explain that they do not tessellate. Explain that hexagons are not the only shape that tessellate.</p>
Slides 15-20 25 minutes (55)	<p>Tessellations Regular Tessellation activity – <i>pencil, paper and ruler, or polydrons</i></p> <p>Optional – <i>online nrich tessellation tool</i></p> <p>Investigating angles</p> <p>Tessellations in art</p>	<p>Explain to the students that the hexagons in beehives are regular hexagons, which means that their sides are all equal length, and the angles inside are all equal too. Talk through the regular shapes from triangle up to octagon.</p> <p>Ask the students if they know the names of any other regular shapes with more than 8 sides. Introduce the tessellation exploration activity, where the students will be using paper, a pencil, and a ruler to test whether each of the regular shapes tessellate. They can do this by attempting to draw the same shape starting from each edge of their original shape, or by using polydron kits, if available.</p> <p>Explain that if the shapes overlap, or leave a gap, then that shape does not tessellate. The students should do up to pentagon, and if time they can also try out the heptagon and octagon (or higher!).</p> <p>Ask the students which of the regular shapes tessellate – triangle, quadrilateral (or square), and hexagon.</p> <p>If students are able to access a laptop, you may also wish to run this activity online, so that students can explore higher number polygons, using the nrich online tessellation tool, found here: https://nrich.maths.org/interactive-environments/tessellation-interactivity. Unhide slide 18 if using this.</p> <p>Move to slide 19, and explain that there is another way we can investigate which shapes have regular tessellation, by seeing if their interior angle is a factor of 360. Explain the interior angle size for a triangle, square and hexagon, and ask the students how many of each shape could fit around a single point.</p> <p>Move to slide 20, and explain that tessellations can be used to make lots of cool art, including work such as Metamorphosis II, by Escher. Ask the students if they can spot the bee in the artwork.</p>

Slides & Time	Overview	Activity (see script for further details)
Slide 21-22 10 minutes (65)	Semi-regular tessellations <i>Pencil, paper and ruler, polydrons, or nrich tool.</i>	Explain that as well as regular tessellations, there are also semi-regular tessellations, which achieve the same tiling pattern with no gaps or overlaps, but by using two or more different shapes together instead of one. They may have seen this when they were drawing octagons, as the shape that appears in the middle is a square. Ask the students to try creating one of these semi-regular tessellations, by using pencil, paper and rulers, polydrons, or the online nrich tool. Go to slide 22 and explain that tessellations with more than one shape are rare in nature, because they are very complex. Give pineapple skin and the giant's causeway as examples of not quite perfect semi-regular tessellations found in nature.
Slide 23 10 minutes (75)	Break	Drinks and biscuits and comfort break.
Slide 24 10 minutes (85)	How much can each shape fit? <i>Worksheet 2, counters / jellybeans</i>	Explain that each of the shapes on their worksheet have the same perimeter. They may measure to check if they wish. Ask the students to try to fit as many counters inside the outline of each shape, reducing gaps, and ensuring they don't go outside the line. They should count up the number in each shape, and see that the amount the hexagon can fit is more than the square, which is more than the triangle. An example for this activity can be found here: https://youtu.be/CEmYXYpAhvk?si=csuJ4y4kc5pj7dZw&t=30
Slides 25-26 5 minutes (90)	Behind the beehive Video Clip 3 The strength of hexagons	Play the Behind the Beehive video clip to summarise the students learning so far, also found at https://www.bbc.co.uk/programmes/p00jfltj . Move to slide 26 and explain that humans also use hexagons to build lots of things, since they are strong and efficient. Talk through the examples of honeycomb panels in airplanes, mirror backing on the James Webb Space Telescope, crash structures in F1 cars, and honeycomb packaging in cardboard boxes. Ask the students if they know any other examples.
Slide 27 10 minutes (100)	Bubbles into hexagons! Demonstration or student activity	Explain that honeycomb doesn't actually start out as a hexagon – they start out as a cylindrical wax cells, and pressure and heat mould them into hexagonal prisms. This activity may be run as either a demonstration or a full class activity. Demonstrate, using a water-washing up liquid mix, a straw, and a plate, how spherical bubbles form a hexagonal bubble when 7 are blown all touching each other. Explain that the beehive cells are not made of bubbles, so they go through a slightly different process to form hexagonal prisms, which they will investigate now. You can also find an example of this activity here: https://youtu.be/TRr_PSotawY?si=MpL6Xrv0OssyHLKO&t=129

Slides & Time	Overview	Activity (see script for further details)
Slides 28 - 29 10 minutes (110)	Cylinders to a hexagonal prism Playdoh hexagons activity	<p>Handout playdoh to each of the students and instruct them to make 7 small spheres or roughly equal size. Ask the students to place the spheres into a typical flower shape, with one in the middle and the other six touching around the outside. Tell them to press gently on the top of the spheres using the palm of their hands, until the gaps between the spheres have disappeared. Ask them what has happened to the central sphere – it should now be a hexagon!</p> <p>You may wish to get the students to make lots of spheres, and place them together in groups or as a class, repeating the process to form lots of hexagonal shapes, just like honeycomb.</p> <p>Go to slide 19 and show the graphic of a large pattern of circles all together, that form a pattern of hexagons when put under pressure, and use it to demonstrate how the hexagonal honeycomb pattern forms.</p>
Slide 30 10 minutes (120)	Further Activities Feedback, tidy up, questions time Ask the Ri	<p>Give a brief re-cap of the session.</p> <p>Don't forget to collect any questions which arise, and email them to us: masterclasses@ri.ac.uk</p> <p>We will send you answers as soon as possible. Then these can be reported back to the students at their next Masterclass session. In this way you cannot be "caught out" by a question. It also demonstrates the point that not everything in maths is known, but some questions need time and research to find answers sometimes, and sometimes the answer has not been found by anyone yet, of course! Maybe our Masterclass students will be the ones who solve the problem when they are older?</p>
Slide 31 10 minutes	Extension: Hidato Beehive Worksheet 3	<p>Bees are all about efficiency? Have a go at this extra challenge, focusing on a different type of efficiency – paths!</p> <p>Following the instructions on your sheet, work out the order in which the bee must visit each honeycomb cell to make it from the start to the finish. The bee may not visit any cell more than once.</p>
Slide 32 10 minutes	Extension: How much wax Worksheet 4	<p>We heard earlier that a bee typically uses 6 grams of honey to make 1 gram of wax. But how does this play out when creating patterns of different shaped cells.</p> <p>Using the instructions on the worksheet, work out how much wax each tessellation pattern would need to build the wax cells. Note that the way the 12 shapes are layout out will impact how much wax is needed – what would happen if the shapes were all lined up in one line?</p>