Masterclass Session Script

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This icon refers to the supporting slide in the presentation.

These icons indicate there is an activity to do, or a worksheet to complete.

This icon indicates there is a video to watch.

The Mathematics of Bees OTS Masterclass	Slides/ Activity
Introduction (10 minutes) Welcome to today's Masterclass. In this session, we'll be exploring the mathematics of bees, and working out the hidden science behind their beehives. While you're waiting for us to get started, can you guess what the zoomed in images on your worksheet show? They're all things that appear in the natural world!	1
We'll start by watching a clip all about shapes in nature – Clip 1: Nature's Shapes	
We have 9 close up images of shapes in nature. For each one, can you guess what the zoomed-out image shows. If you finish super quickly, as an extension you could turn your worksheet over and use the space to name or describe the different shapes each image shows. Let's go through the answers! • **What does image 1 show?**	2
Image 1 shows a turtle shell! The patterned plates on the turtle shell's surface are usually six sided, making them hexagons! The shape of the plates on their shells helps them to be strong and durable (long lasting) and allow them space to grow. • **What about image 2?**	3
Image 2 shows some dew drops on a dandelion. The dew drops are spherical, and this is because of something called surface tension. Because the dew drops are touching something, in this case the seed heads of the dandelion, they naturally form a shape that gives it the smallest surface area possible. This shape is a sphere! • **What is image 3 showing?**	4
Image 3 shows a leaf! Leaves come in lots of shapes and sizes, and the cells, or building blocks, of the leaves form a very intricate pattern. In 2024 a new group of shapes have been investigated and named, called "soft cells". They are typically curved and have faces that are not flat, as well as being able to form a pattern without gaps (something we'll investigate later in the session). These soft cell shapes are often found within leaves, and are a very common shape found in nature. • **What does image 4 show?**	5
Image 4 shows a goat's eye. The black part of the eye is called the pupil, and goats are special as their pupil is a wide rectangular shape! This is very useful, as it gives them a really wide field of view, nearly 340° panoramic vision. This means that they can spot predators even when grazing with their heads down, a navigating terrain and because their pupil can take in lots of light, they have very good night vision. • **What about image 5?**	6
Image 5 shows the circular rings on a tree stump. These rings indicate how much the tree has grown over time. Each ring typically indicates a year of growth, so you can work out the age a tree was by counting the number of rings on its stump! The width of the	7

ring can also tell us the conditions the tree was growing in that year – a wider ring means it grew more, so it could indicate good conditions, whereas a narrow ring could indicate poor conditions, like drought.

- **What does image 6 show?**

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Image 6 is a zoomed in view of a dragonfly's eye. Dragonflies have two compound eyes, made of up to 30,000 hexagon shaped lenses each, which gives then nearly 360° vision.

- **What about image 7?**

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Image 7 is the inside of okra, a type of fruit! This one might be quite tricky if you haven't heard of okra before! Okra grows into a long, ridged, five-sided pod, and when you cut a small part of it off, you can see the pentagonal (5 sided) structure clearly! This shape gives Okra 'fivefold symmetry', which means it will look the same every time you rotate it 1/5 of a full circle. Lots of other things in nature have fivefold symmetry, like some flowers, and starfish!

- **What do you think image 8 shows?**

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Image 8 is of some very cool volcanic rock, specifically made of basalt. Basalt rock forms from lava, and when the lava cools down it can form these hexagonal columns. There are lots of famous examples of this, like the Giants Causeway in Ireland.

- **And last but not least, what do you think image 9 shows?**

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Image 9 shows poop! Specifically, wombat poo, which has a very distinct shape. It comes out in neat cubes and is the only animal we know that produces cubic poop. The wombats' intestines have walls with changing levels of stiffness, so as the poo makes its way through it is shaped into a cube. This helps it to be stacked, so they can use their poo to mark their territory.

So, we have lots of shapes within nature, all giving different animals, plants and rocks special features or evolutionary advantages. Nature doesn't choose shapes at random. These shapes solve problems. But what shapes do bees use? Today we're going to find out**!**

Bees – what do we know? (10 minutes)

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We know lots of shapes appear all over the natural world, but today we're going to focus on the shapes that help bees. But first, I want to know…

- **What do you already know about bees?**

We know that bees are pollinators. They like to go from flower to flower collecting pollen and nectar, and as they do this, the pollen sticks to their feet and helps the plants they visit pollinate, leading to new seeds or fruit being produced.

We also know that some bees make honey, though not all bees do! In fact, there are over 20,000 species of bees! Some live in large groups of other bees, called colonies, and some live by themselves. It might surprise you that over 90% of the world's bee population are called solitary bees, or bees that live by themselves in soil, trees or the stems of plants.

The bee species that do produce honey, typically live in large colonies inside beehives. The honey they produce is stored in honeycomb, which they make from wax they also produce. These beehives are built with a very specific shape design – the hexagon!

- **What do we already know about hexagons?** They have six sides!

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A perfect, or regular hexagon, has six sides all the same length as each other. The hexagonal wax cells in bee hives are used to store honey and raise larvae, or young bees.

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Every beehive, all over the world, all has the same internal structure – they are made up of honeycomb wax that forms small hexagonal cells.

But why do all the bees from different areas of the world make the exact same shape? Why aren't they made up of other shapes, like triangles or diamonds?

- **We call the 2D version of the shape a hexagon, but what do we call a 3D version of a shape with a hexagon base?** A hexagonal prism!

Honey Hunters in Nepal (10 minutes)

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How do bees build these structures? Let's have a look at this clip!
In the clip, Brian Cox explains that for every gram of wax a bee produces and uses to build their honeycomb in the hive, they will have to use more than six grams of honey. This means bees need to build efficient hives, using as little wax as possible. If bees waste wax, they waste honey. If they waste honey, the colony might not survive. Today we are going to see investigate how bees do this to ensure their survival.

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- **If you were designing a home to house thousands of bees and lots of honey, what would you need to consider when designing the size, layout and shape of the wax cells?** Discuss with the person next to you.

Bees need honeycomb that is strong and sturdy, fits together with no gaps, stores lots of honey, uses as little wax as possible, and are all roughly the same size, so a honeybee could fit inside.

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If wax cells leave gaps, that's wasted space and wasted wax. Hexagons form a tessellation pattern, This means that the shape can be repeated into a tiled pattern, without any gaps or overlapping.

Only a few shapes can tessellate. Circles and spheres, for example, have lots of gaps when they tile together. These gaps are wasted space, and bees are all about efficiency. However, the hexagons aren't the only shapes that can tessellate. Let's discover what those shapes are!

Tessellations (25 minutes)

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The hexagons the bees use are regular hexagons, and this means that all six sides are the same length, and all the angles inside the hexagon are the same.

We're going to investigate some other regular shapes – a triangle that has three equal sides, a square with four equal sides, a pentagon with 5 equal sides, a heptagon with 7 equal sides, and an octagon with 8 equal sides.

- **Do you know the names of any regular shapes with more than 8 sides?**

For each of these shapes, we are going to investigate whether they tesselate or not – remember, that means if they can tile together without overlapping or leaving gaps. Using plain paper, draw a pattern of triangles, squares and pentagons, and work out if they tessellate. As an extension, try the heptagon and octagon! You may also wish to use polydrons here.

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From this activity we can see that only three regular shapes tessellate on their own – the hexagon, the triangle, and the square! So, bees had these three choices.

So is there another way we could quickly work out whether a shape tessellates or not? Yes! We could look at the size of their interior angle.

A regular shape will only tessellate if the size of their interior angle is a factor of 360. Pentagons have an interior angle of 108 degrees, so there is no way to place them next to each other, without creating a gap, or having overlap – they are not a factor of 108 is not a factor of 360!

Equilateral triangles have interior angles of 60 degrees.

- **How many triangles will fit together around one point?** 360 / 60 = 6. So 6 triangles can fit together.

Squares have interior angles of 90 degrees.

- **How many squares will fit together around one point? 360** / 90 = 4. So 4 squares can fit together.

Hexagons have interior angles of 120 degrees.

- **How many hexagons will fit together around one point?** 360 / 120 = 3. So 3 hexagons can fit together.

Tessellations can also be seen in famous artwork, such as Metamorphosis II, made by the Dutch artist M. C. Escher in 1940, which was carved into a piece of wood. It depicts many different tessellations and cool illusions appear.

- **Can you spot a bee in the artwork?**

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Semi-regular tessellations (10 minutes)

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As well as regular tessellation, there are also semi regular tessellations. These involve two or more shapes placed together to make a no gap or overlap pattern. You can mix shapes to tessellate, but bees don't. There are 8 semi-regular tessellation patterns, which you can see here.

You may have spotted this if you drew an octagon.

- **What shape appears in the middle of the octagons?** A square!

Using pencil and paper, polydrons, or the nrich online tessellation tool, test out some of these tessellations! Try to see if you can create one of these tessellations!

- **If you were a bee, would these be easy to build?** No!

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As it happens, tessellations with more than one shape in are very rare in nature, because they require very complex, detailed patterns to form.

Some examples of tessellation patterns in nature that have more than one shape in, include pineapple skin, and the Giant's causeway (made of the basalt rocks we saw earlier in the workshop). These aren't perfect semi-regular tessellations, but they give us an example of how complex it would be for the bees to make the wax cells multiple different shapes.

How much can each shape fit? (10 minutes)

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We now know that hexagons are the most structurally stable of the tessellating shapes, providing a strong structure for the bees to build their hives. But does that make the hexagon the best choice? What else do bees need to consider? How much honey each wax chamber can hold!

We are now going to compare the efficiency of each shape, by investigating how much honey each shape could hold, if it took the bees the same amount of wax to build.

Let's take the triangle first. It has a set perimeter – 30cm. Let's try and fill it with counters/jellybeans and count the number it takes to fill the triangle up. We do not want any counters/jellybeans to overlap or go over the perimeter line.

Okay so we now know that a triangle of 30cm perimeter takes around 18 jellybeans. Let's transfer those counters/jellybeans to a square with the same perimeter and you should notice that there is a gap! You can add more counters/jellybeans. For the same amount of wax perimeter, the square can hold, around 23 jellybeans, so therefore it can hold more honey. Continue with your investigation with the other shapes on the worksheet, and record how many counters/jellybeans each shape takes. Compare the number of counters/jellybeans in the hexagon to the other shapes. It should be able to hold around 26 jellybeans. From this we can determine that the hexagon is the most efficient shape for honey storage as it provides the most volume for honey, compared to how much wax it has taken to build it.	
Behind the Beehive (5 minutes) Let's watch a clip that summarizes what we've learnt about the bees and their efficient tessellating shape. Nature isn't the only place hexagons can be found! Humans copied the bees, because hexagons are strong *and* efficient. Clockwise, each image shows: -Honeycomb panels in aircraft -James Webb Space Telescope mirror backing -Formula 1 crash structures -cardboard honeycomb packaging • **Can you think of anything else where you've seen hexagons used?**	25 26
Bubbles into hexagons (10 minutes) We know now that hexagons are the most efficient shape, and we've explored how they have inspired us to create lots of strong, efficient things! But, what we haven't looked at is how these shapes actually form. It may surprise you that the bees don't actually make the hexagons, they make lots of little circular wax cells instead, and let physics do the hard work!. So how do we get from circles to hexagons? To demonstrate this, we'll need to use something that is very famous for being circular – how about a bubble! As I blow these bubbles onto the plate, the begin to touch. When all seven bubbles are touching, the middle bubble forms six sides- turning into a hexagon! The bees' hexagonal cells aren't made of bubbles though, so the process they undergo to form hexagonal prisms from spheres is a little bit different! Let's investigate!	27
Spheres to a hexagonal prism (10 minutes) So how do these circular wax cells in the beehives actually form hexagons? With pressure! We are going to make our own hexagonal structures in a similar way to bees, but we will be using playdoh instead of honeycomb. For this, you will need to make 7 spheres of playdoh. Try to make them all the same size and as circular as possible. You then need to arrange the playdoh in a flower shape, with one sphere one in the center and 6 surrounding it. You might notice that there are lots of gaps between the balls, making this a very inefficient use of space! Using the palm and	28

ball of your hand, push down on all the spheres to get rid of the gaps between them. Try to get them as flat as possible. • **When you remove your hand, what do you notice about the middle sphere?** It should naturally shape into a hexagonal prism. Let's see this on a larger scale! Take your playdoh and shape it back into spheres. In your group, work together to place as many spheres next to each other as you can and press down on them. What pattern do we notice now? More hexagonal prisms are appearing! But why do these structures form this way? Because of pressure! So we can see how lots of cylinders all together, with pressure and heat from the honeycombs, will form our fantastic, efficient hexagonal honeycomb shape – how fantastic!	29
End of session – recap and feedback (10 minutes) Thank you very much for joining this masterclass today, we hope you enjoyed it and learnt a lot about data science in sport! If you would like to have a go at the extension activities, we have a worksheet on batter recruitment available, or you could try out the activities using a different sport, like golf, tennis, hockey, and more! If you have any questions, comments or thoughts we would be happy to hear them now. You can also ask the Ri, by emailing any questions to masterclasses@ri.ac.uk.	30
Hidato Beehives (10 minutes) Bees are all about efficiency! Have a go at this extra challenge, focusing on a different type of efficiency – paths! Following the instructions on your sheet, work out the order in which the bee must visit each honeycomb cell to make it from the start to the finish. The bee may not visit any cell more than once.	31
How much wax? (10 minutes) We heard earlier that a bee uses 6 grams of honey to make 1 gram of wax. But how does this play out when creating patterns of different shaped cells. Using the instructions on the worksheet, work out how much wax each tessellation pattern would need to build the wax cells. Note that the way the 12 shapes are layout out will impact how much wax is needed – what would happen if the shapes were all lined up in one line?	32