

Maths of Voting Masterclass:

Extra Background Information

Voting is a key aspect of any democratic society; it allows for all individuals to have their opinions and preferences taken into account when deciding on key issues, such as who will run the country, or major decisions to be made (e.g., the referendum to leave the European Union).

Voting is also a key part of many forms of entertainment. In competition shows, the viewing public is able to vote for their favourite contestant to boost their chances of winning. In so-called 'social deduction' video and board games, players can vote for who they believe to be a saboteur within the group in order to complete a main objective.

There are many different voting systems, and each can sometimes produce a different winner from the exact same votes. Mathematics play an integral part in this, as votes can be formulated/counted differently based on the system and can be utilised as a tool to help win elections. This raises the dilemma of what makes a voting system fair or unfair.

Arrow's Impossibility Theorem

Proven by Kenneth Arrow, who won a Nobel Memorial Prize for his work, Arrow's impossibility theorem highlights that an ideal voting system is seemingly impossible to achieve. It states that a clear order of preferences in a vote cannot be determined without also breaking at least one mandatory principle of fair voting.

In theory, a fair voting system should follow the principles below:

- Pareto/unanimity – if every voter shares the same preferences, then that top preference should win
- Universality – there should be one clear winner
- Independence of irrelevant alternatives (IIA) – the order of preference between options A and B should depend only on A and B
 - In other words, the preference between A and B should not change based on the order of preference between A and C, for example
 - If A is voted more preferred than B, then B should not become more preferred than A if C is eliminated

Arrow's impossibility theory finds that the only system that meets all these principles in a scenario where voters are given more than two candidates to vote on, is a dictatorship, in which only one person's vote is counted.



Kenneth Arrow. Image Credit: Linda A. Cicero via Stanford

Voting Systems

First Past the Post

First Past the Post (FPTP), also called first-preference plurality (or simply plurality) has been used in British General Elections since the Middle Ages. Voters select their first preference only, and the winner is the candidate with the highest number of votes (plurality), even if they do not have over half of the votes (a majority).

FPTP has been abandoned as a voting system by several countries, including Australia and New Zealand, for several reasons. Although FPTP is simple, it is vulnerable to vote-splitting i.e., when there are two very similar (or “cloned” candidates that, even though the support for their general ideology/policies is the most popular, because voters can only select one or the other, both lose to a less popular candidate with no “clones”. Similarly, FPTP is vulnerable to “lesser evil voting”, in which a voter is forced to vote for a candidate that is not their favourite, to stop a greater evil from winning. This commonly takes the form of tactical voting in British General Elections.

Alternative Vote

Alternative Vote (AV) was adopted in Australia in 1918-19 as a replacement to First Past the Post. In AV, voters select their preferred candidates in order, and can choose to vote for as many candidates as they like. If a candidate has an immediate majority (over 50% of the number one votes), they win. If not, the candidate with the fewest votes is eliminated, and ballots with that candidate as their first preference are changed to the second preference, rather than being discarded. This is repeated until a candidate has a majority.

This system aims to counteract vote splitting, because it allows voters to choose several preferred candidates rather than just one. AV also tends to favour more broadly liked candidates, as more extreme/polarizing candidates who receive fewer first-preference votes are typically eliminated first. AV is a good example of how mathematics – i.e., the way that votes are counted and processed, can favour certain types of candidates.

In 2011, a Referendum was held in the UK to vote on whether our system should be changed to Alternative Vote, but 67.9% of voters voted against this.

Borda Count

Borda Count, named after French mathematician Jean-Charles de Borda, was devised by him in 1770. Borda Count is rarely used in electoral systems, although it is used in Slovenia to elect seats reserved for ethnic minorities. Borda Count works by having voters order candidates by preference, with points then awarded according to that ranking. In its purest form, a point is awarded to each candidate for every other candidate they have beaten. For example, the lowest preference candidate scores no points on that ballot, the second-to-lowest scores 1, and so on. The candidate with the most points after all ballots are counted, wins.

A heavily modified version is used by Eurovision Song Contest to decide the winner. Each country has 5 industry professionals who can rank their top ten performances. Their first preference is awarded 12 points, second is awarded 10, and then subsequent preferences are awarded 8-1 points. The public can also call in to vote, and these votes are then tallied with 12 more points awarded to the most voted, 10 to the second, and so on.

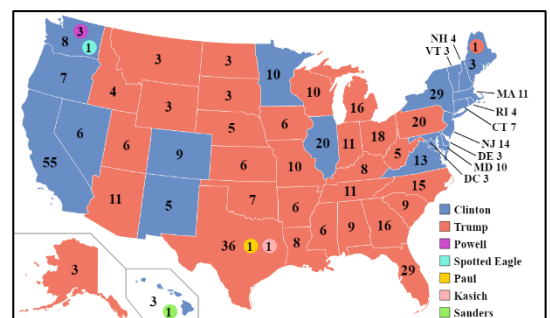
Despite a Borda Count ballot being filled in much the same way as Alternative Vote, the mathematics behind the way votes are counted can produce very different results. Borda Count can allow for a candidate that is everyone's middle choice to gain more points than a candidate that has the majority of first-place votes but nearly as many last-place votes. In a system that allows for more than one candidate per party, a party may enter any number of candidates in an election to guarantee points and increase the odds of a win for the party – this is because Borda requires voters to rank all candidates, while AV allows voters to rank as many (or as few) as they like.

Electoral College

In the United States, a Presidential Election is held every four years. The millions of citizens who vote (called the popular vote) do not *strictly* vote for the President, but instead for their state's group of electors. These electors have pledged to vote for their party's candidate, and if they win the popular vote in their state, they will all vote for their pledged candidate in the electoral college.

Each state has a number of electors proportional to the population of that state, and there are 538 electors total – meaning that 270 electoral votes are needed for a candidate to win the Presidency. This system partly originated from the people who wrote the US Constitution not having trust in the common voter.

On most occasions, the winner of the popular vote ends up also winning the electoral vote. However, there have been several occasions where the candidate who loses the popular vote wins in enough states to send more electors to the electoral college. In recent memory, this includes George W. Bush's victory over Al Gore in 2000, and Donald Trump's victory over Hillary Clinton in 2016.



2016 electoral college map. Image Credit: Gage via Wikimedia Commons