Choose your lottery numbers

On your bit of paper write the 6 numbers you would choose if you were playing the lottery…

The rules:

- There are **59 numbers (integers from 1 to 59)** to choose from.
- You **choose 6** with the aim of matching the 6 numbers that are drawn in the lottery, to win the jackpot.

Royal Institution Primary Maths Masterclasses

Off the shelf Masterclass:

Get Lucky!
Probability and the Lottery.

rigb.org
@Ri_Science

Image credits: Pixabay

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The Royal Institution

Our vision is:
A world where everyone is inspired to think more deeply about science and its place in our lives.

Image credits: Tim Mitchell

Royal Institution activities

- Online videos & activity resources
- National education programmes
- Membership
- London-based:
  - Talks and shows
  - Holiday workshops
  - Family fun days
  - Faraday Museum

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Image credits: The Royal Institution, Paul Wilkinson, Katherine Leedale

The CHRISTMAS LECTURES

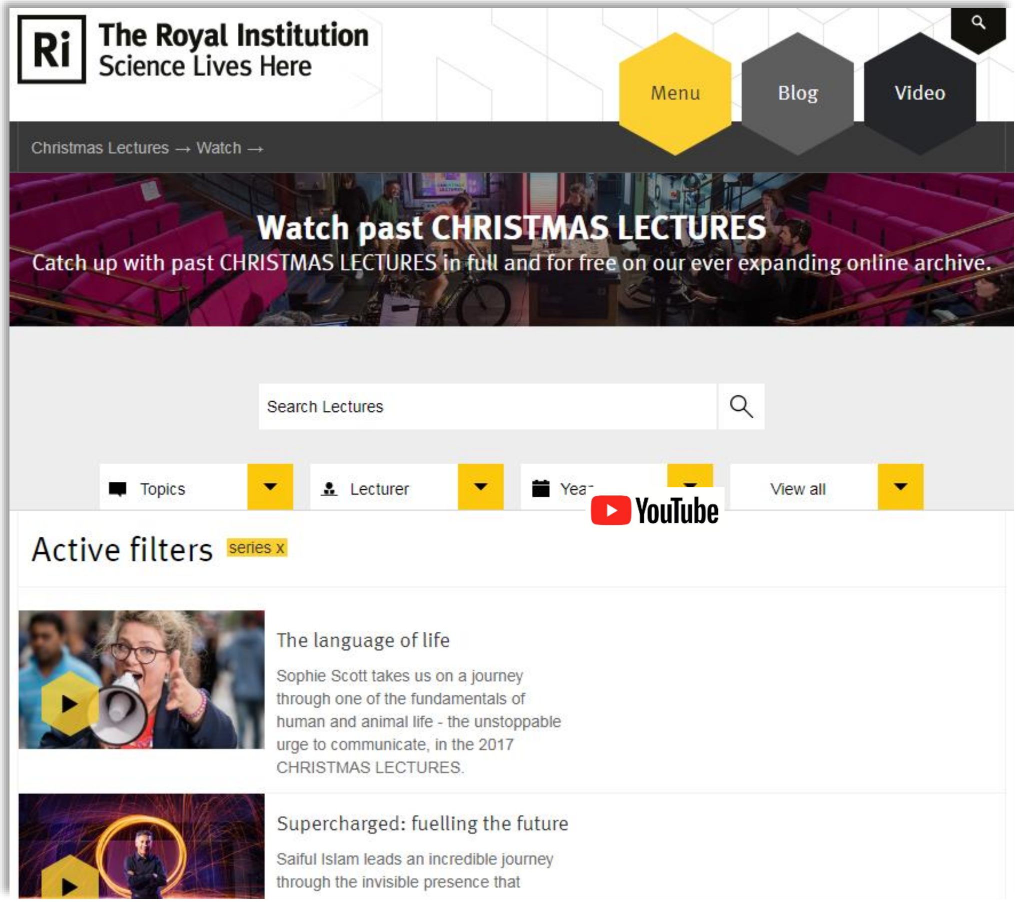
The CHRISTMAS LECTURES are the Ri's most famous activity and are televised on the BBC. The first maths lectures by Prof. Sir Christopher Zeeman in 1978 started off the Masterclass programme!

Christmas Lecturers include Michael Faraday, David Attenborough, Carl Sagan, Richard Dawkins, Alison Woollard, Saiful Islam & Alice Roberts

Image credits: Tim Mitchell, Paul Wilkinson

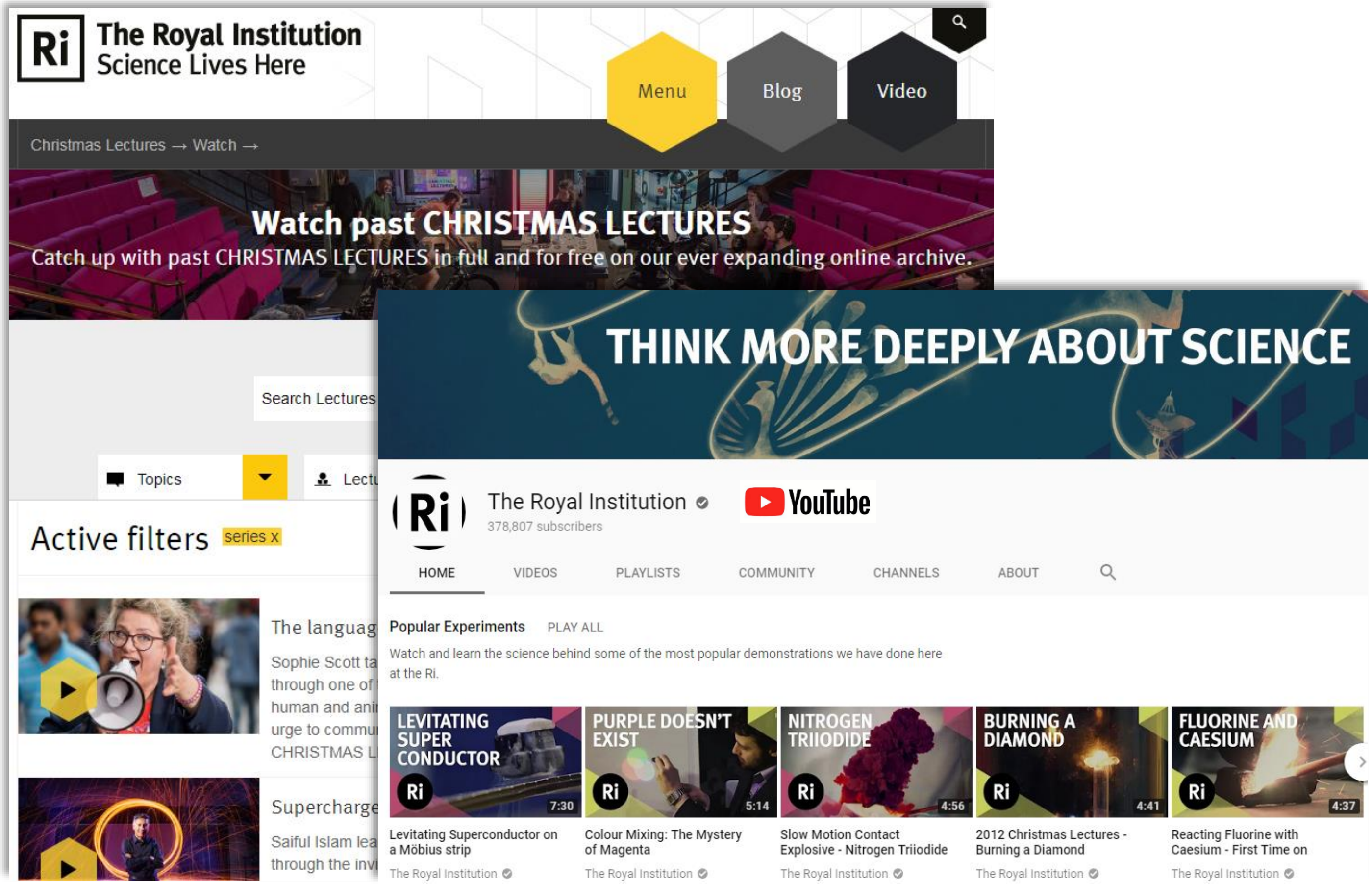
Royal Institution videos

- CHRISTMAS LECTURES – on the Ri website



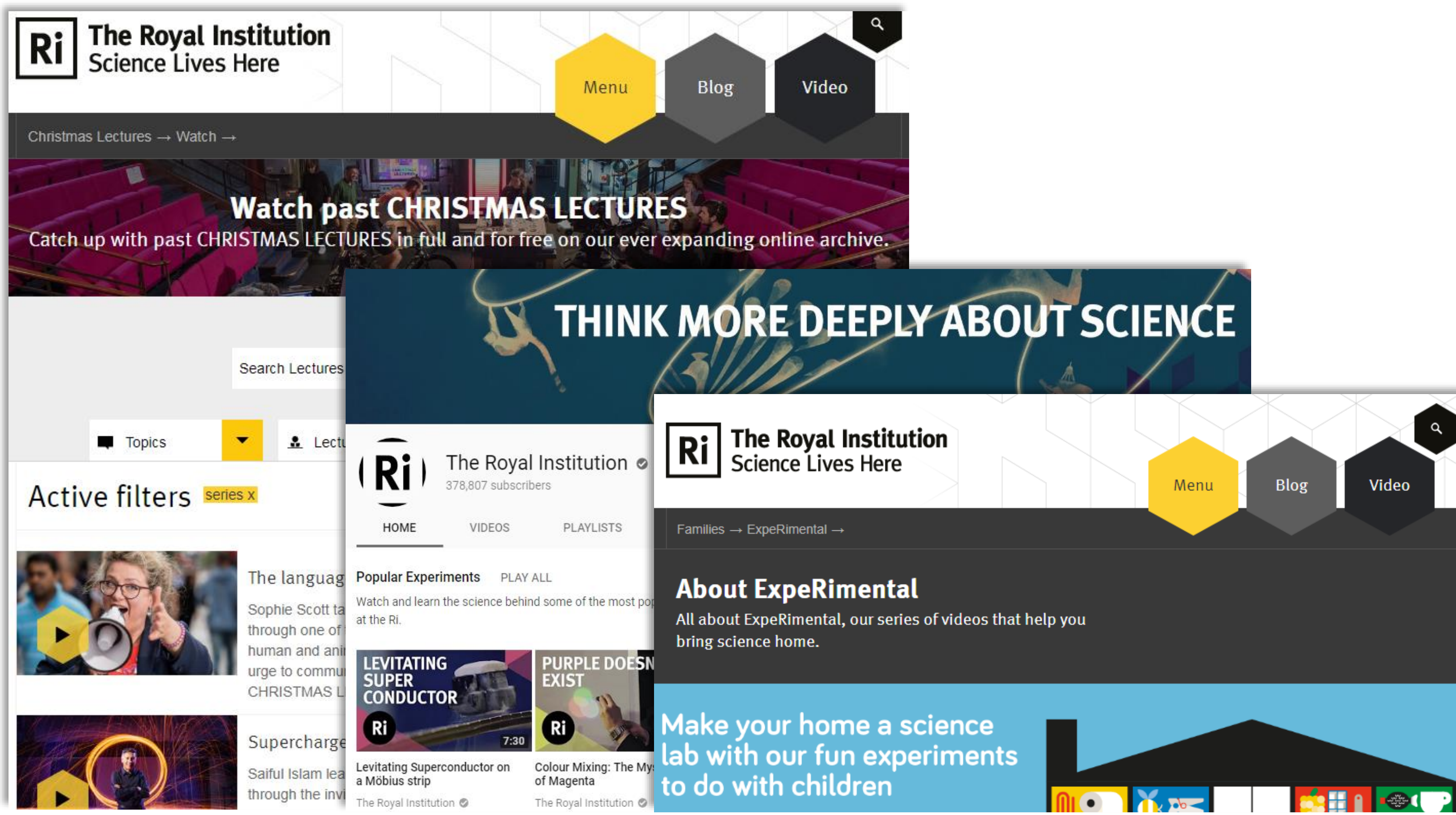
Royal Institution videos

- CHRISTMAS LECTURES – on the Ri website
- Ri on YouTube – experiments, videos & talks for all ages



Royal Institution videos

- CHRISTMAS LECTURES – on the Ri website
- Ri on YouTube – experiments, videos & talks for all ages
- ExpeRimental – science experiments at home



Get Lucky!

Probability and the Lottery

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One Coin Flip

H T

$$\frac{1}{2}$$

Two Coin Flips

You could have…

T T

What other combinations could you have?

Two Coin Flips

T T	T H
H T	H H

Three Coin Flips

TTT TTH
THT HTT
THH HTH
HHT HHH

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Is it worth playing the lottery?

What do we need to find out?

Combinations

Mini Lottery

Players choose two numbers from 1,2,3,4,5

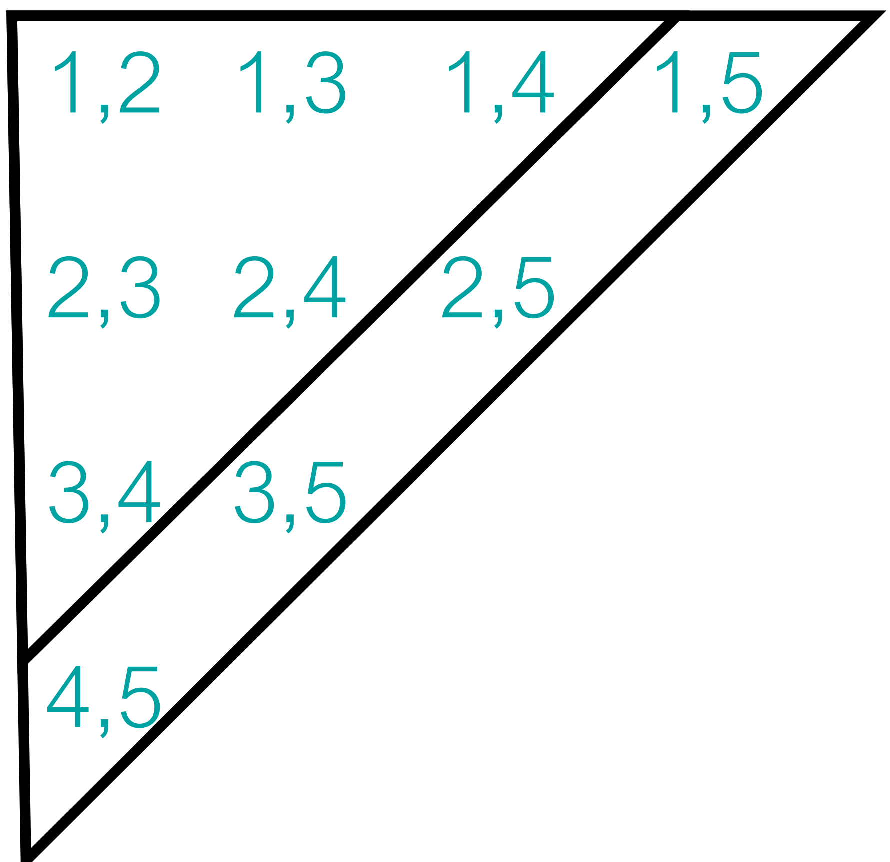
If you choose the two numbers that are chosen from the machine, then you win the jackpot!

a) Write out ALL the possible pairs of numbers you can choose. Order doesn't matter in the lottery!

b) Finished? Write out ALL the possible groups of three numbers you can choose from the five.

Mini Lottery

Write out ALL the possible pairs of numbers you can choose!



$${}_5C_2 = 10$$

$${}_4C_2 = 6$$

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Mini Lottery

Three numbers chosen from 1,2,3,4,5

Write out ALL the possible groups of three numbers you can choose from the five

3,4,5   2,4,5   2,3,5   2,3,4

1,4,5   1,3,5   1,3,4

1,2,5   1,2,4

1,2,3

$$ {}_{5}C_{3} = 10 $$

		How many lottery numbers you choose. **CHOOSE**					
How many numbers to choose from. **FROM**		**0**	**1**	**2**	**3**	**4**	**5**
	0						
	1						
	2						
	3						
	4			6			
	5			10	10		

		How many lottery numbers you choose. **CHOOSE**					
How many numbers to choose from. **FROM**		**0**	**1**	**2**	**3**	**4**	**5**
	0						
	1						
	2						
	3						
	4			6			
	5			10	10		

		How many lottery numbers you choose. **CHOOSE**					
		0	**1**	**2**	**3**	**4**	**5**
How many numbers to choose from. **FROM**	**0**						
	1						
	2						
	3						
	4			6			
	5			10	10		

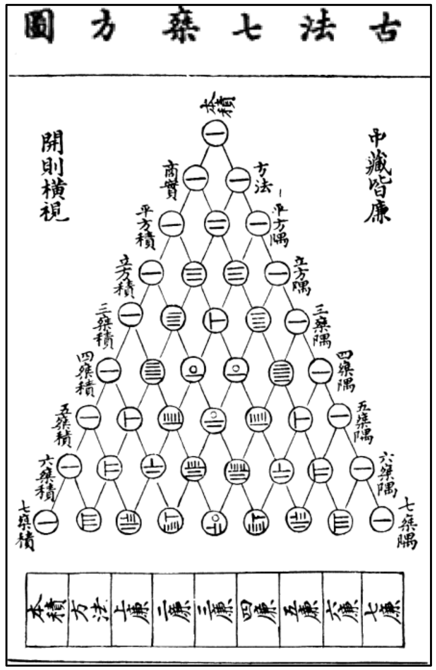
		How many lottery numbers you choose. **CHOOSE**					
		0	**1**	**2**	**3**	**4**	**5**
How many numbers to choose from. **FROM**	**0**						
	1						
	2						
	3			3			
	4			6	4		
	5			10	10	5	

		How many lottery numbers you choose. CHOOSE					
		0	**1**	**2**	**3**	**4**	**5**
How many numbers to choose from. FROM	**0**						
	1		1				
	2		2				
	3		3	3			
	4		4	6	4		
	5		5	10	10	5	

		How many lottery numbers you choose. **CHOOSE**					
		0	**1**	**2**	**3**	**4**	**5**
How many numbers to choose from. **FROM**	**0**						
	1		1				
	2		2	1			
	3		3	3	1		
	4		4	6	4	1	
	5		5	10	10	5	1

		How many lottery numbers you choose. **CHOOSE**					
How many numbers to choose from. **FROM**		**0**	**1**	**2**	**3**	**4**	**5**
	0	1					
	1	1	1				
	2	1	2	1			
	3	1	3	3	1		
	4	1	4	6	4	1	
	5	1	5	10	10	5	1

Pascal’s Triangle



Also known as

Yanghui Triange

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		How many lottery numbers you choose. **CHOOSE**					
How many numbers to choose from. **FROM**		**0**	**1**	**2**	**3**	**4**	**5**
	0	1					
	1	1	1				
	2	1	2	1			
	3	1	3	3	1		
	4	1	4	6	4	1	
	5	1	5	10	10	5	1

Pascal’s Triangle

Can you fill in the rows below?

1

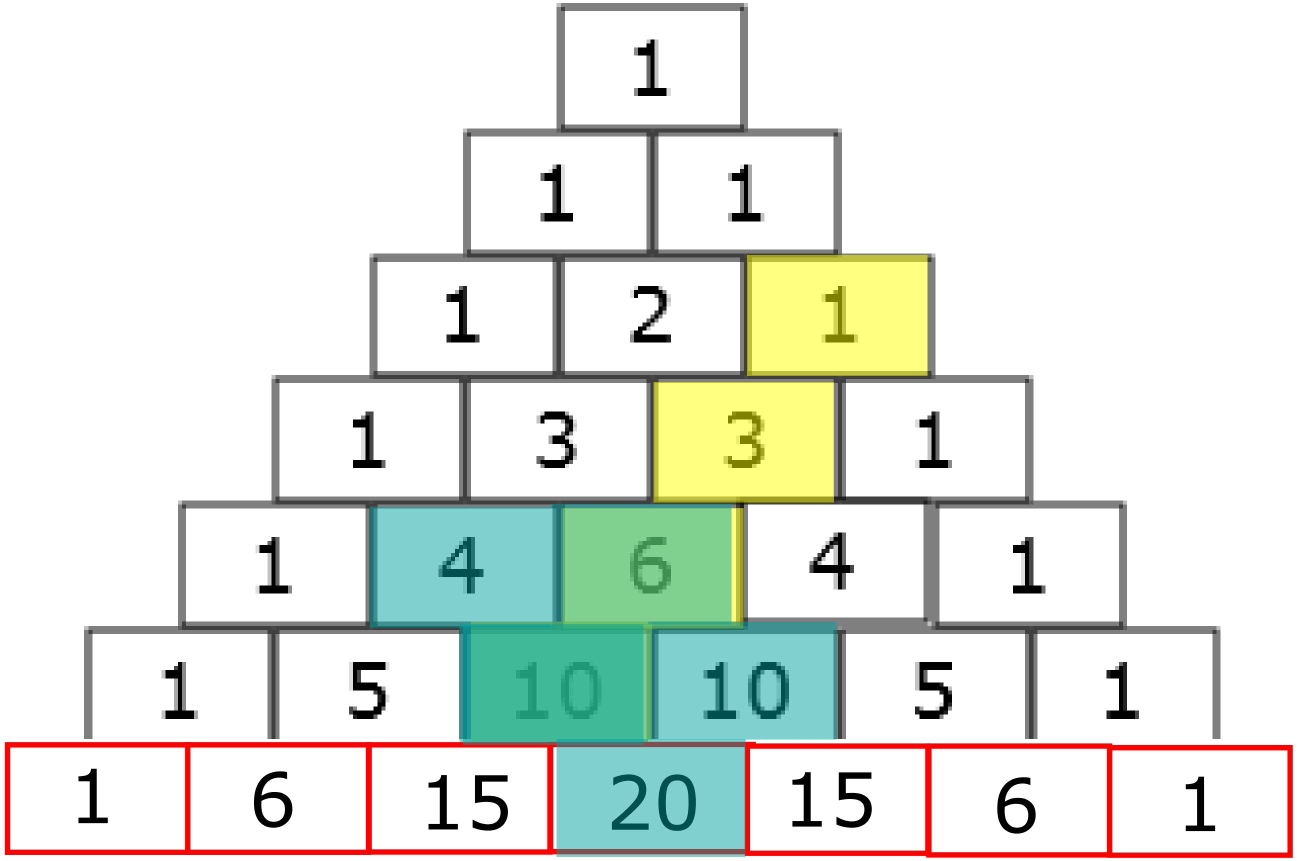
1 1

1 2 1

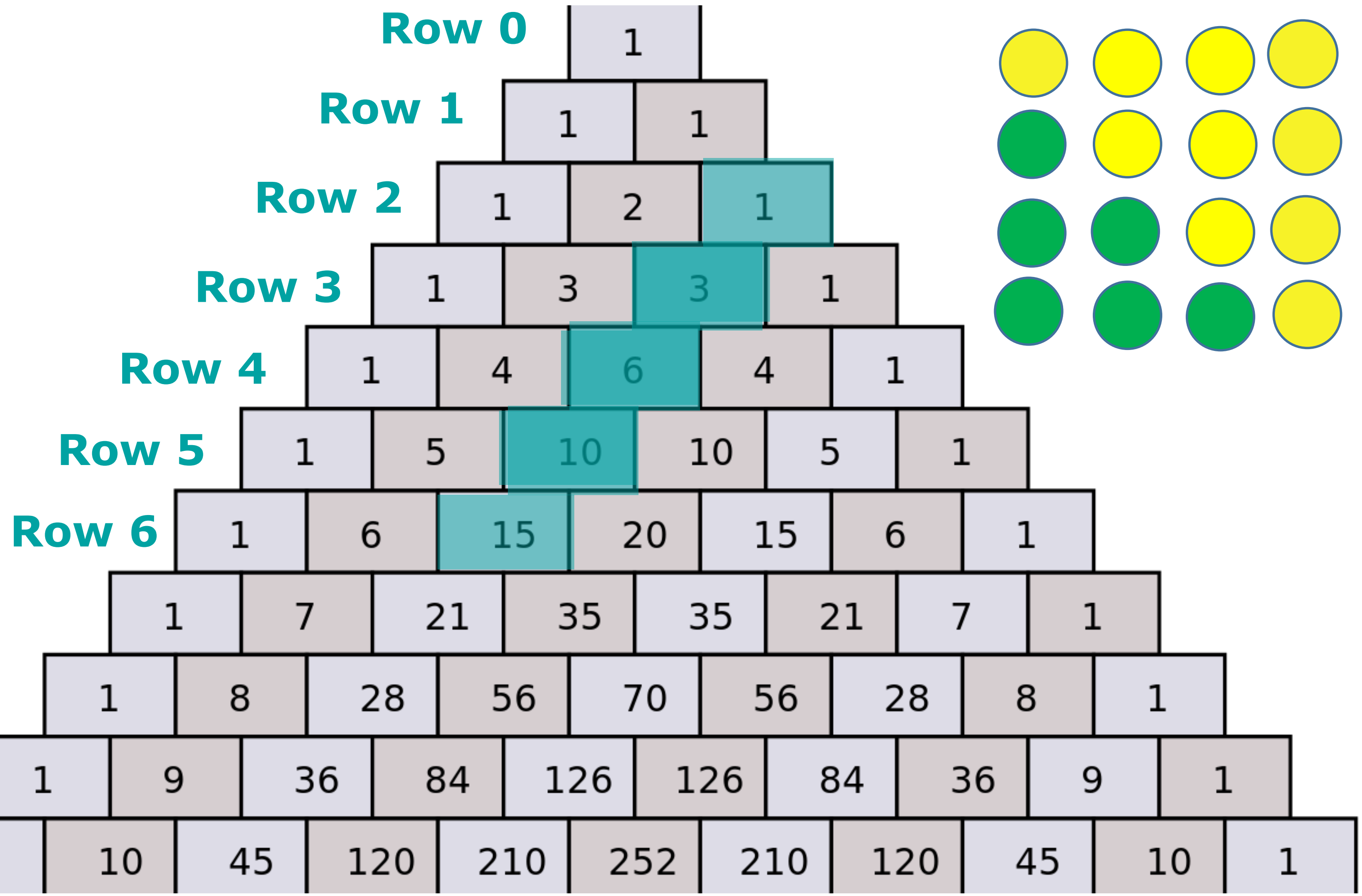
1 3 3 1

1 4 6 4 1

1 5 10 10 5 1


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Pascal’s Triangle



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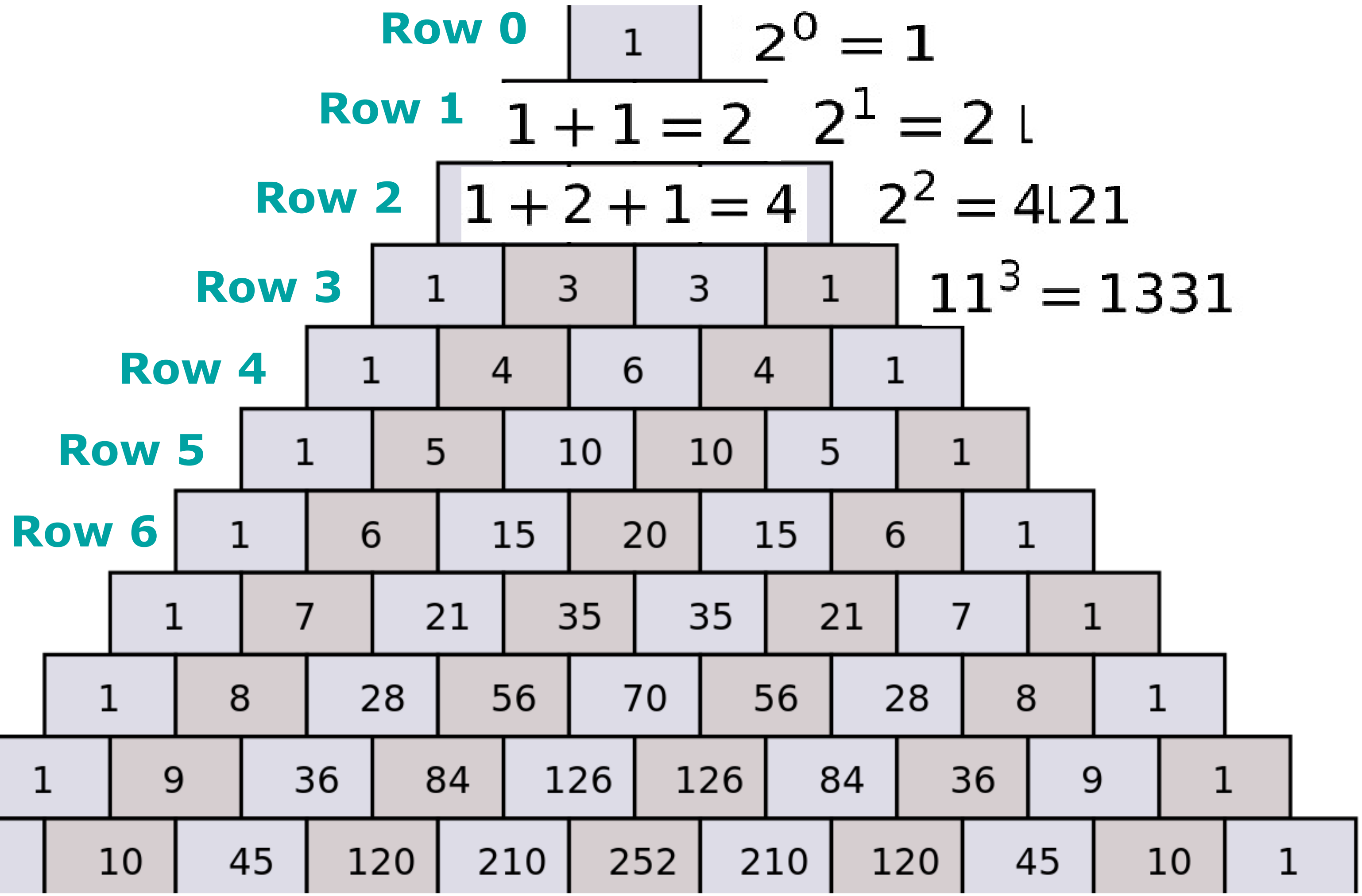
Credit: Nonenmac at English Wikipedia

Pascal's Triangle



Credit: Nonenmac at English Wikipedia

		How many lottery numbers you choose. **CHOOSE**					
How many numbers to choose from. **FROM**		**0**	**1**	**2**	**3**	**4**	**5**
	0	1					
	1	1	1				
	2	1	2	1			
	3	1	3	3	1		
	4	1	4	6	4	1	
	5	1	5	10	10	5	1

Row 0 1

Row 1 1 1

Row 2 1 2 1

Row 3 1 3 3 1

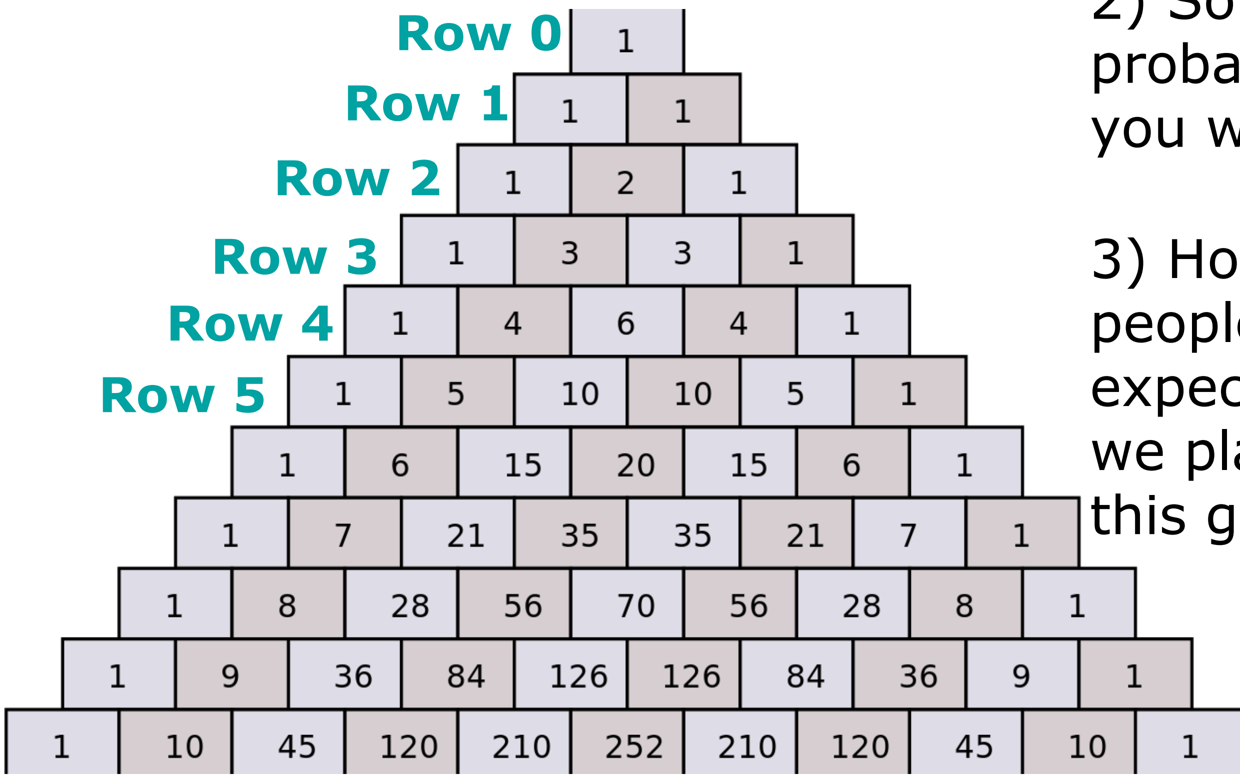
Row 4 1 4 6 4 1

Row 5 1 5 10 10 5 1

Mini Lottery

Players choose two numbers from 1,2,3,4,5,6,7

1) How many different groups of numbers are there that you could choose?



2) So what is the probability that you will win?

3) How many people would we expect to win if we played with this group?

Credit: Nonenmac at English Wikipedia

Lottery - how many ways?

Interactive Pascal's Triangle

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Lottery - how many ways?

Two cells above 59 C 6 in Pascal’s triangle…

4582116	**40475358**

45057474

So is it worth it?

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Lottery - how many ways?

$$= 45,057,474$$

$$= 1 \text{ in } 45,057,474$$

$$= \frac{1}{45,057,474}$$

$$= 0.00000002219$$

What does that mean…?

You need to play **45,057,474** times to expect to win once

1. How many years does it take to play that many times, if you buy a ticket a week?

2. How many lifetimes is that?

3. If £2,000,000 is the average jackpot winnings per person, how much money do you expect to lose in total (if you could play for that many lifetimes)?

Finished?

Can you think of another way of calculating the probability of winning the jackpot?

What does that mean…?

1. How many years does it take to play that many times, if you buy a ticket a week?

45,057,474 / 52 = 866,489.88

2. How many lifetimes is that?

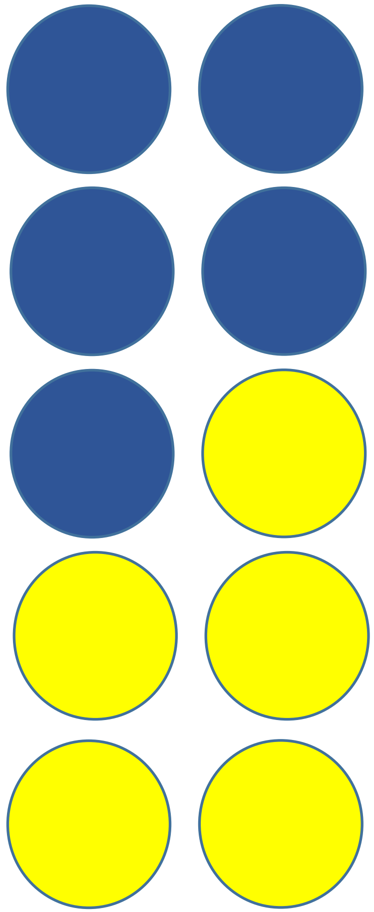
866,489.88 / 74 = 11709.32

3. If £2,000,000 is the average jackpot winnings per person, how much money do you expect to lose in total (if you could play for that many lifetimes)?

45,057,474 x 2 – 2,000,000 = 88,114,948

Extension Material: Expected value of a lottery ticket.

Extension activity: Is it worth it?



Game 1

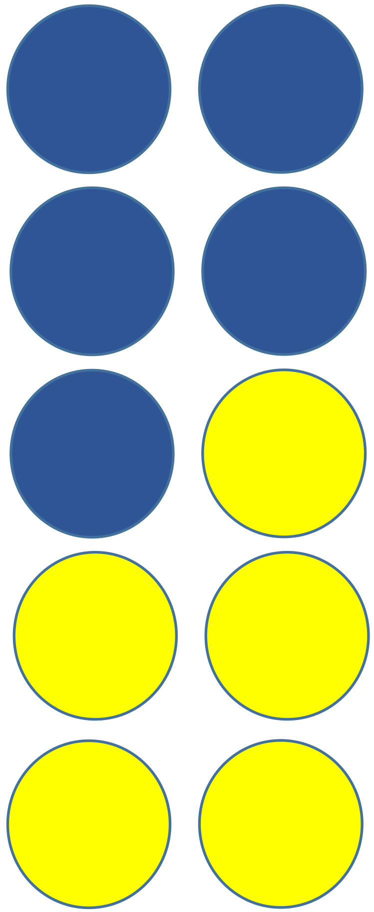
Cost 3 sweets to play

Win 10 sweets if you draw blue.

Win nothing if you draw yellow

Is it worth playing?

Is it worth it?



Game 2

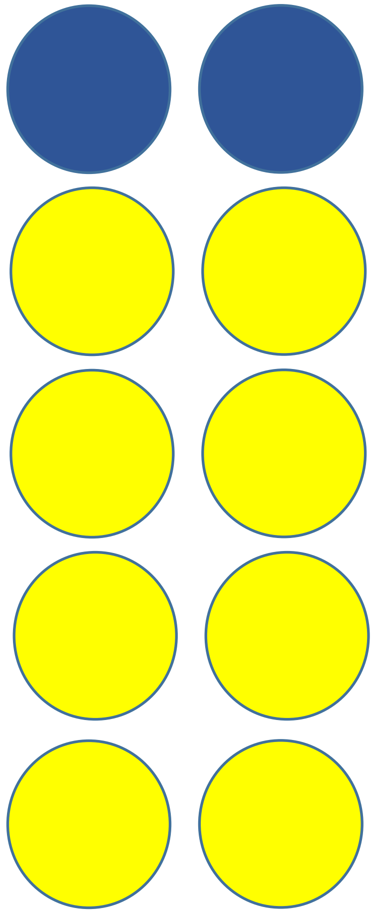
Cost 3 sweets to play

Win 5 sweets if you draw blue.

Win nothing if you draw yellow

Is it worth playing?

Is it worth it?



Game 3

Cost 3 sweets to play

Win 10 sweets if you draw blue.

Win nothing if you draw yellow

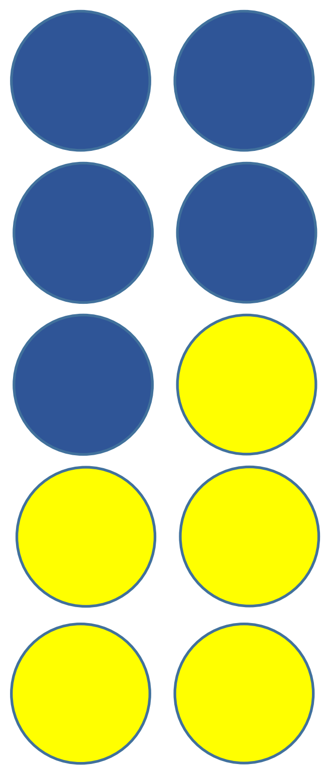
Is it worth playing?

Expected value

Expected value = probability win x prize if you win

= 0.5 x 10

= 5



Game 1

Cost 3 sweets to play

Win 10 sweets if you draw blue.

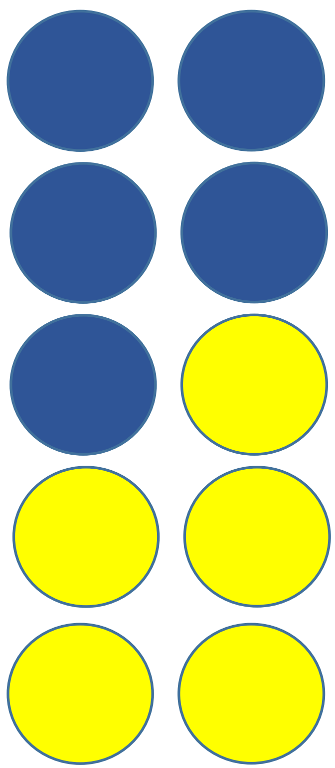
Win nothing if you draw yellow

Is it worth playing?

Expected value

Expected value = probability win x prize if you win

$= 0.5 \times 5$
$= 2.5$



Game 2

Cost 3 sweets to play

Win 5 sweets if you draw blue.

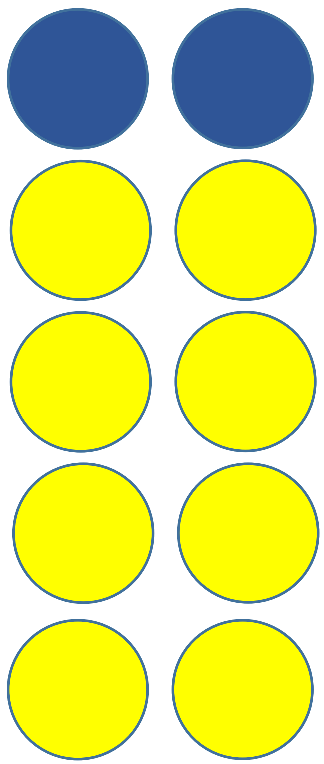
Win nothing if you draw yellow

Is it worth playing?

Expected value

Expected value = probability win x prize if you win

= 0.2 x 10
= 2



Game 3

Cost 3 sweets to play

Win 10 sweets if you draw blue.

Win nothing if you draw yellow

Is it worth playing?

Expected value

Expected value =

probability you win x prize if you win

+

probability that you lose x prize if you lose

$$E(x) = P_w \times X_w + P_L \times X_L$$

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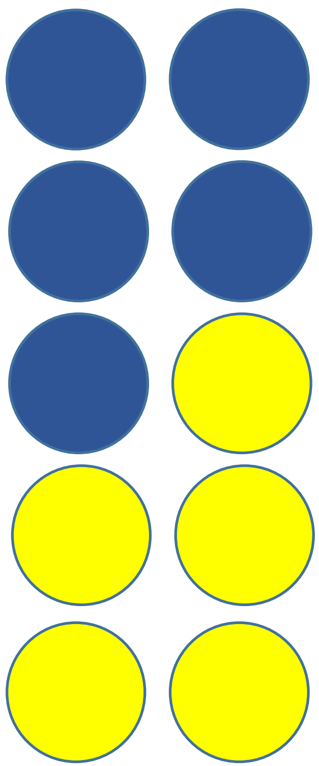
Expected value

$$E(x) = P_W \times X_W + P_L \times X_L$$

$$E(x) = 0.5 \times 10 + 0.5 \times 0 = 5$$



Game 1

Cost 3 sweets to play

Win 10 sweets if you draw blue.

Win nothing if you draw yellow

Is it worth playing?

Expected value of a lottery ticket

$$E(x) = P_1X_1 + P_2X_2 + P_2X_3 + \ldots\ldots P_nX_n$$

Number of numbers matched	Outcomes (X) (Average prize money per person)	Probability (p)	Prize multiplied by probability
6	£2,000,000		
5 and bonus	£50,000		
5	£1000		
4	£100		
3	£25		
	TOTAL:		

Expected value of a lottery ticket

$$E(x) = P_1X_1 + P_2X_2 + P_2X_3 + \ldots\ldots P_nX_n$$

Number of numbers matched	Outcomes (X) (Average prize money per person)	Probability (p)	Prize multiplied by probability
6	£2,000,000	0.00000002219	
5 and bonus	£50,000		
5	£1000		
4	£100		
3	£25		
	TOTAL:		

Expected value of a lottery ticket

$$E(x) = P_1X_1 + P_2X_2 + P_2X_3 + \ldots\ldots P_nX_n$$

Number of numbers matched	Outcomes (X) (Average prize money per person)	Probability (p)	Prize multiplied by probability
6	£2,000,000	0.00000002219	
5 and bonus	£50,000	0.00000013316	
5	£1000	0.0000069244	
4	£100	0.00045874742	
3	£25	0.01039827488	
	TOTAL:		

Expected value of a lottery ticket

$$E(x) = P_1X_1 + P_2X_2 + P_2X_3 + \ldots..P_nX_n$$

Number of numbers matched	Outcomes (X) (Average prize money per person)	Probability (p)	Prize multiplied by probability
6	£2,000,000	0.00000002219	£0.044
5 and bonus	£50,000	0.00000013316	£0.007
5	£1000	0.0000069244	£0.007
4	£100	0.00045874742	£0.046
3	£25	0.01039827488	£0.260
	TOTAL:		

Expected value of a lottery ticket

$$E(x) = P_1X_1 + P_2X_2 + P_2X_3 + \ldots..P_nX_n$$

Number of numbers matched	Outcomes (X) (Average prize money per person)	Probability (p)	Prize multiplied by probability
6	£2,000,000	0.00000002219	£0.044
5 and bonus	£50,000	0.00000013316	£0.007
5	£1000	0.0000069244	£0.007
4	£100	0.00045874742	£0.046
3	£25	0.01039827488	£0.260
	TOTAL:		**£0.36**

On average, for every £2 spent on a ticket –
maths says you should expect to get 36p back.

So is it worth it?

Expected value of a lottery ticket

There's never been a better day to play the lottery, mathematically speaking

Buying a ticket for today's national lottery draw makes mathematical sense for the first time in its history.

The jackpot - which will be around £58m - is the largest since the lottery began in 1994. But that's not what makes today's draw unusually interesting.

Today's draw is noteworthy because the rule changes made to the lottery last year have resulted in an anomalous situation where the "expected value" of each £2 ticket - that is, the amount you can expect to win per ticket on average - is more than £2.

January 2016

Credit: Alex Bellos, The Guardian

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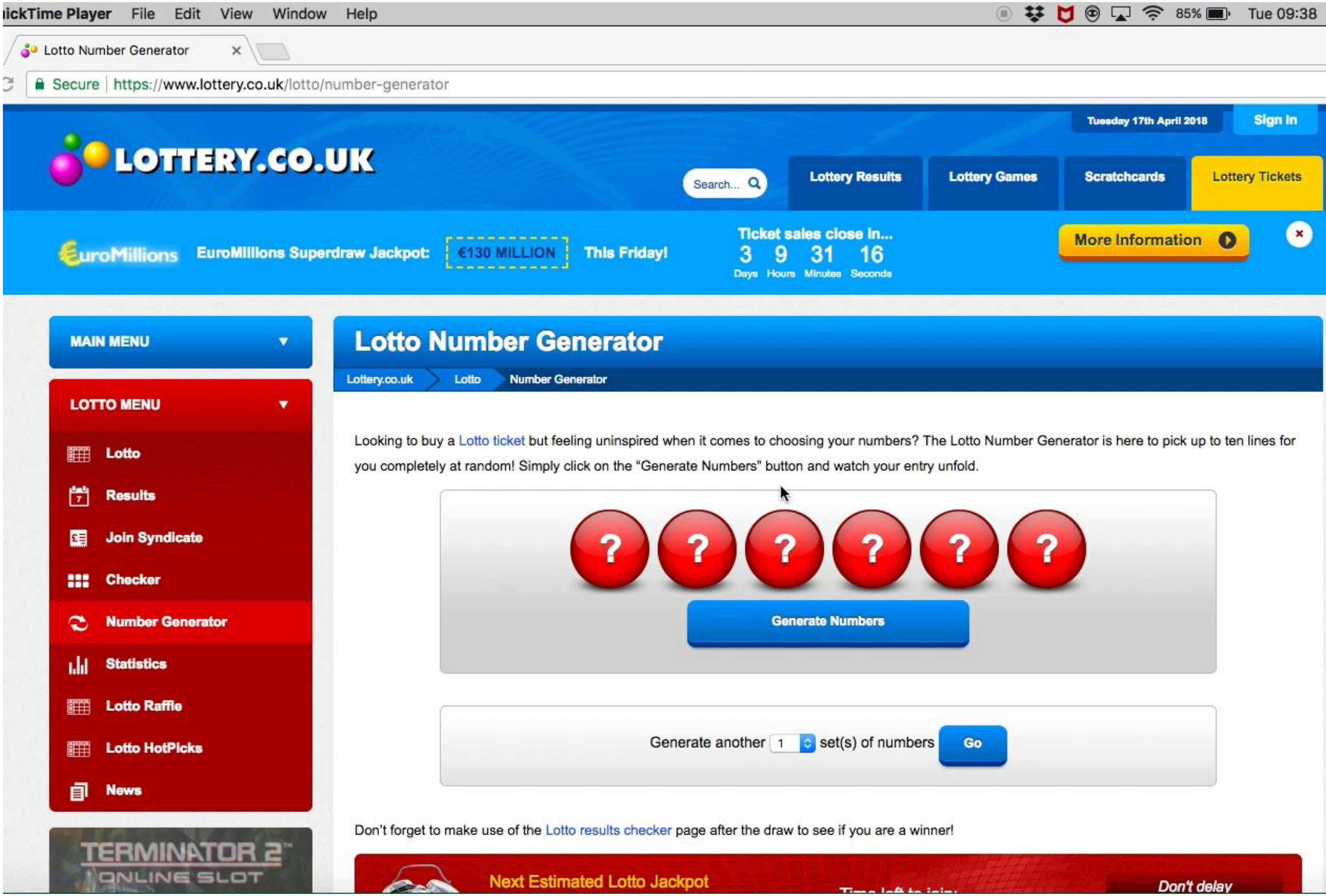
Lets play the Lottery!

https://www.lottery.co.uk/lotto/number-generator



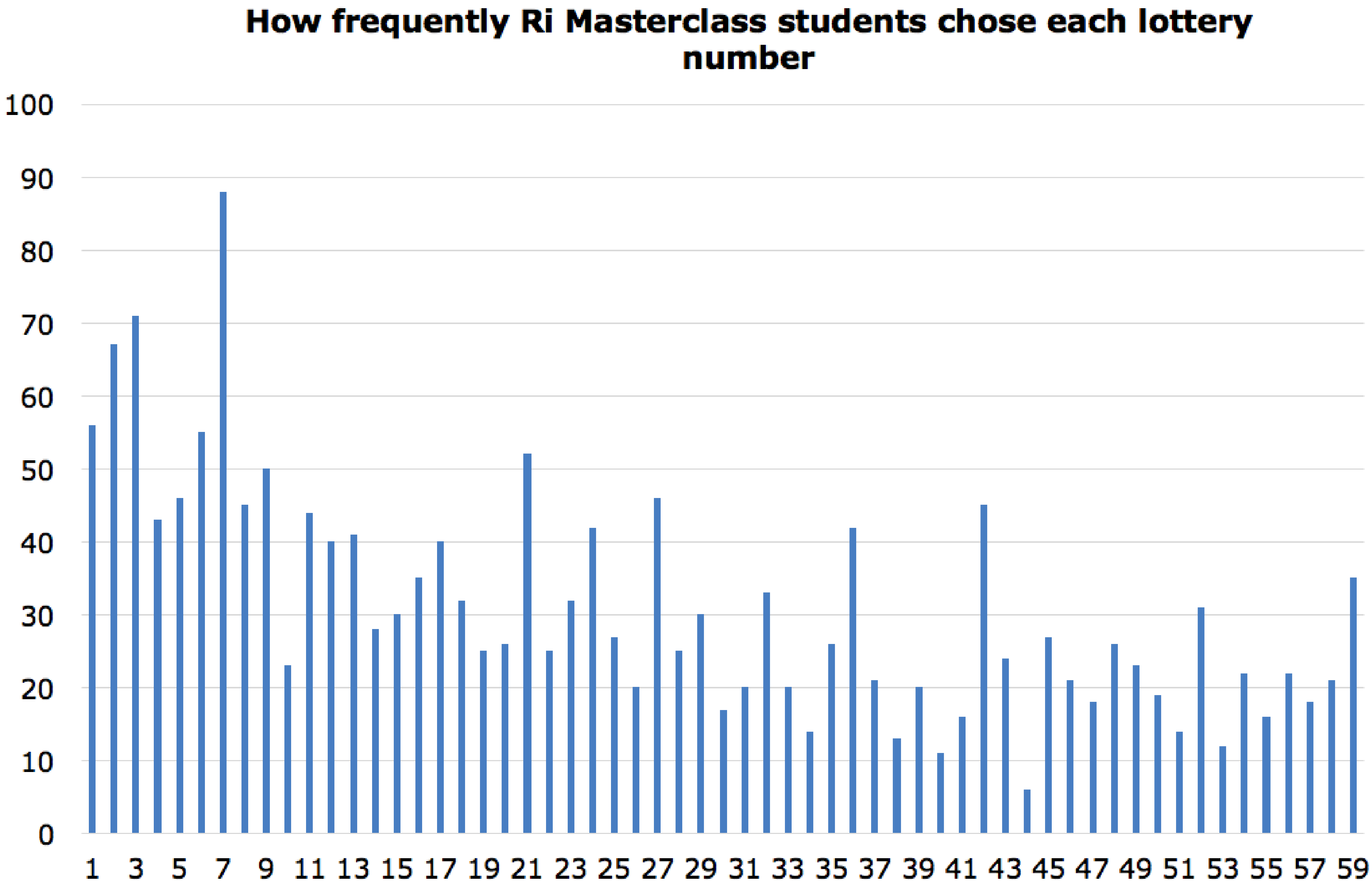
Which numbers would you choose?

A) 7, 17, 26, 31, 49, 56

B) 1,2,3,4,5,6

C) 3, 7, 11, 14, 27, 30

D) 36, 37, 39, 40, 52, 53


How frequently Ri Masterclass students chose each lottery number
100
90
80
70
60
50
40
30
20
10
0
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59

Popular numbers

23 March 2016, Lottery Results

14 21 42 35 07 41 43

No. of matches	No. of winners	Prize per winner
Match 6	0	£0
Match 5 + bonus	6	£10,016
Match 5	4,082	£15
Match 4	7,879	£51
Match 3	114,232	£25

We hope you have enjoyed exploring the maths of the National Lottery with us!

What questions do you have?

Any unanswered questions can be written down and emailed to “Ask the Ri Masterclass Team” using this email masterclasses@ri.ac.uk

We don’t know all the answers instantly, but we will find out and get back to you before the next Masterclass.

Any comments you have about what you enjoyed or what you’d like to do more of can be written on the post-it note and handed in.

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Go further: combinations and thinking systematically

Small Change https://nrich.maths.org/754

Stage: 3 ★ ★ ★

In how many ways can a pound (value 100 pence) be changed into some combination of 1, 2, 5, 10, 20 and 50 pence coins? Remember, the aim is not just to get the answer but to find a good method and to explain it well.

Greetings https://nrich.maths.org/615

Stage: 3 ★

There are 30 students in a class and it is found that in any subset of 4 students from the class each student has exchanged Christmas cards with the other three. Show that some students have exchanged cards with all the other students in the class. How many such students are there?

Image credits: XXXXX

Go further: Pascal's Triangle

Investigating Pascal's Triangle https://nrich.maths.org/5593

Stage: 2 and 3 ★ ★

I think that it's time to look at Pascal's Triangle afresh. So, let's see what happens when we turn it around in a special way.
So we start with the layout as usual, turn it anticlockwise 45 degrees and then take off the top line of ones.

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

1	1	1	1	1
1	2	3	4	
1	3	6		
1	4			
1				

1	2	3	4
1	3	6	
1	4		
1			

Image credits: XXXXX

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Thank you!