## Digital Computers

This icon means there＇s a slide，or slides in the presentation to accompany this line of the script．


This icon indicates the students will have an activity to do，or something to write．

## Introduction（10 minutes）


［This next section can be skipped if this masterclass is not the first one in the series．］

These masterclasses are organised by the Royal Institution． Has anyone heard of the Royal Institution before？It was founded in 1799，and has always been about letting everyone
皿-穼 have access to science－organising masterclasses and lectures，including the Christmas lectures．

Many famous discoveries have been made in the Faraday building where they are based，including 10 chemical elements．Michael Faraday，who the building is named after，did work there on electricity and optics，which we all use every day．

The Ri are perhaps most famous for Christmas Lectures for young people，which have taken place since 1825．They have been televised for several decades，and many past series are available on the Ri website．The Masterclass programme was born out of the Christmas lectures delivered by Christopher Zeeman in 1978，which were the very first ones on mathematics！

There are many opportunities for you to visit the Ri building in London, and see the historic rooms for yourself. There is a small museum too. There are lots of talks and holiday events for young people: all the details are on the Ri's website.

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## Counting and fingers (20 minutes)

You might be wondering why this workshop is called 'digital computing', and you're counting on your fingers. Does anyone know why? Another word for a finger is a 'digit'. This isn't a coincidence! The early ways people used to write numbers were designed to look like fingers one, two and three, as you can see here. And if you rotate this one around, you can see that the modern symbols also have a connection to the older ones. People in history have always used their fingers to count and calculate, so the word 'digit' means both a finger and a number.

It's also part of other words too - we use the word 'fist' to mean a closed hand, which has five fingers on - but the words 'fist' and 'five' have similar starts. This is true in many other languages too - here you can see that in Dutch and Polish that the words are similar. Does anyone know how to pronounce these? Does anyone know any other examples?

In this workshop we're going to see some ways to use your fingers to count and make calculations. Does anyone know their nine times table? Here's a trick you can use to work out multiplying by nine very easily!

## - Put your hands up in front of you, looking at your palms.

If you want to work out nine times something between 1 and 10 , you simply need to put down the finger corresponding to that number, and the number of fingers either side of the gap will be the two digits of the result. For example, if you want to know $3 \times 9$, you put down the third finger from the left - in this case your middle finger on the left hand - and you see 2 fingers to the left of the gap, and 7 to the right. So $3 \times 9=27$.

## - Take some time to practise this. Here are some examples you can try to work out!

Can anyone suggest why this trick works? The pattern in the multiples of 9 means the tens digit goes up by one each time, and the units digit goes down by one. This is because adding 9 is the same as adding 10 and subtracting 1. The gap in your fingers moves along one each time as a result.

Here's another trick which is slightly harder, but lets you do more different multiplications. First, you need to number your fingers 6
 - 10 as shown on this slide, on each hand.

Now, if you want to multiply two numbers together, you just need to touch together the corresponding fingers to make a bridge.

- First, you count the number of fingers below, including the ones in the bridge. Multiply this by ten, as this will be your tens digit. In this example, we're doing $8 \times 9$ - so if we count the fingers below
including the touching ones, there's 7 , so our tens digit is 7 .
- Next, count the number of fingers on each side at the top. This will give you two numbers, which you need to multiply together to give the units digit. In this example, above the bridge we have 2 and 1 , so $2 \times 1=2$, and that means the units digit is 2 . So the answer is 72 .

So remember, you count the numbers beneath, multiply by ten, and then count the numbers above and multiply them together.


Take some time to practice this, and try the examples on this slide.

What do you think of this trick? It's a lovely method but maybe it might be easier to just remember your times tables...

## Mind Reading trick (25 minutes)

Next we have a really impressive trick.

Each of you take one of these 'mind reading cards', and think about your birthday - specifically the day of the month on which you were born; for example, my birthday is the $\qquad$ th of [month], so I would be thinking of the number __. It should be a number between 1 and 31, and you'll find it might be on some of the cards - maybe on more than one, and maybe even on all of them, but look at each card and decide which cards your number is on, and which it's not on.

Now, I'd like a volunteer to have your mind read. Can someone tell me which cards their birthday is on? I can work out that your birthday is on the $\qquad$ th. Am I correct?
[Using the numbers in the top left of the cards, you can add together the powers of two, 1, 2, 4, 8 and 16 if those cards contain their birthday number. For example, if they tell you their birthday is on cards 0,2 and 3 you'd add $2^{0}+2^{2}$ $+2^{3}=1+4+8=13$. You can try a few examples. If you get one wrong, it might be because they've misunderstood, so make sure they haven't missed any cards.]


I'm not going to tell you how this trick is done, but I want you to think about the numbers on the cards. Maybe look at the smallest number on each card. Now I'm going to give you a minute to think about how this trick works. Discuss with the people around you, and see which cards your birthdays are on, or pick other numbers and try them. Can you figure it out? If you do, can you guess what number your friend is thinking of?

Does anyone have an explanation of how you can do this trick? If we look at these first numbers on each card, they have a special pattern in them. Can anyone tell me what the pattern is? They double each time starting from 1 on the right hand card, it doubles to 2 , then 4 , then 8 , then 16 . And these numbers are what you add together to do the trick.

But these numbers are special: you can use them to make other numbers by adding them together - but what's even better is, you never need to use each number more than once. For example, $6=2+$ 4 (and if you look for the number 6, it's on this card with the 2 and this card with the 4, and nowhere else).


But if we want to find a way to make 10 , we could say $10=2+4+4$. But we don't need to use two fours, because the next number along is 8 , which is just two fours - so we could say $10=2+8$ (and you can find 10 on these two cards). The fact that the numbers double mean we
 never need more than one of each.

Now we're going to try a new way of counting, using these numbers. Each of you will need five strips of post-it note wrapped around your fingers and stuck down, and you will need to write
 the numbers 1, 2, 4, 8 and 16 on the front of each finger. (You should do this using the hand you don't write with, or else you should write on the postits before you stick them on your fingers).

Now, to make 1 , you just use the finger with a 1 , then to make 2 you can use the finger with a 2 ; but to count 3 , you need your 1 finger and your 2 finger together.

- See if you can count all the way up - how far can you go?

Once you've gone as far as you can, challenge your friends - say a number, and see if they can put up the right fingers to make it. [Offer 'Counting in binary' cheat sheet to students if needed.]

## Binary and Decimal (25 minutes)

You've had a chance to count using your fingers - all the way up to 31 just using one hand! Now we can use this to write numbers in a new way as well. These worksheets will let you write numbers in a system called binary - it uses the same numbers 1, 2, 4, 8 and 16 and you put yeses and nos, or 1 s and 0 s , instead of putting your fingers up and down.

This first sheet has all the numbers you can make using only 1,2 and 4 . Once you've finished this, here are the answers - and there's a second sheet with bigger numbers, and space for you to try your own examples.

Computers use binary to store and calculate numbers, and they may have heard of binary as being made of 1 s and 0 s - this is why you never need a 2, because these numbers allow you to only use 0 or 1 of each.

- If we wanted to use more than 5 fingers, what might we label the next column to the left?

If we doubled 16, we'd find we have 32, then 64 and so on.
Here's a question for you to work out. If we labelled all the fingers on both hands, what's the largest number we could represent? Try to work it out.

If we used both hands, we'd have all these fingers as well - 32, $64,128,256$ and 512 . If we add all of these together, we'd get a total of 1023. So you can count to over 1000 using only your fingers!

This system - binary - is actually similar to the system we normally use to write numbers, except instead of multiplying by 2 to get the next column heading we normally multiply by 10 , and write numbers as 1 s , $10 \mathrm{~s}, 100 \mathrm{~s}, 1000 \mathrm{~s}$ etc. This system is called decimal. This time we need more than 0 or 1, because we need to make all the numbers in between, so we might
need $0,1,2,3,4,5,6,7,8$ or 9 of each. This is probably connected to the fact that humans have 10 fingers, so this is the natural way to represent numbers.

Some systems use other number bases - hexadecimal is a system some computer systems use which has 16 digits, and the column headings multiply by 16 each time. This means they need more symbols, and they actually use 0-9 and A-F. This means you can write even bigger numbers using fewer digits.


The ancient Mayans used a system which incorporated base 20, called vigesimal, and had 20 different symbols to represent numbers.


## End of session - recap

In this session we've thought about counting on your fingers, and how _ you can use your fingers to count and do calculations using some nice maths tricks. We've also learned how to read people's minds using binary numbers, and compared binary to other systems including our own decimal numbers.

Take your mind reading cards away with you and use them to read people's minds - ask them to think of their birthday, or a number, and then add up the numbers in the corner. Secretly you're thinking like a computer!

