Christmas Lectures 2006

THE NUM8ER MY5TERIES –
Presented by Professor Marcus du Sautoy

LECTURE 5: THE QUEST TO PREDICT THE FUTURE

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Part 1

Marcus Well, welcome to The Number Mysteries. How can I predict the future? Well if time travel was possible, it would be really easy. I could just travel forward a year, see what the lectures are going to be about next year, come back and tell you all about it. But sadly I don't know any way to do time travel, so I'm just going to have to say goodbye to my time machine, Leo, ah. Some people think you can predict the future by looking into the tarot cards. Others say you can predict it by staring into a crystal ball. Yes, the mists are clearing; I can see that the only way to predict the future is: mathematics.

Mathematics is the ultimate fortune teller, it can help you to predict whether the earth is going to be hit by a meteor and how long it's going to take for the sun to keep burning. It also can help you to predict the flight of a football. Now good footballers make scoring goals look really easy. They need to do lots of calculations to predict exactly where the football is going to go. Well to help me explain some of the mathematics of football, I'll like you to give a Royal Institution welcome to, from Chelsea's Ladies Team, Nicky de la Salle. Now, Nicky I must admit that I'm actually an Arsenal fan.

Nicky Really? Maybe I should have walked straight back out?

Marcus No honestly. I'm going to put partisan feelings aside for the night, and in the greater cause of mathematics I'm going to get you to help me to explain a little bit about how Beckham bends his footballs, if we can. But I think the lecture theatre is a little bit small to kick footballs around, as you see, I just took that woman's head out over there. Where do you think we could do this?

Nicky There's a park over the road, if you want to join me out in the cold?

Marcus OK. Well I'm not going to come out in the cold. I'll join you a little later from inside here. So, let's send Nicky out, to the park outside, we'll come back to her a little bit later to see how we can bend this ball. Let's give Nicky a big round of applause, we'll see her later.

The whole story of how we can predict the future actually starts with trying to understand simple objects, like a ball. Now some things are really predictable, if I drop this ball, you all know what's going to happen, but if the ball was a little bit heavier, would it fall faster? Well the ancient Greeks thought it did. After all, if I drop a ball and a feather, the feather takes a lot longer to hit the floor than a ball. So the ancient Greek philosopher, Aristotle, actually thought if something was 10 times heavier, it would actually, would go 10 times faster to the ground. But actually in the early 1600s an Italian professor of mathematics, called Galileo Gallia, decided to investigate the matter. Now we're going to do an experiment which Galileo certainly never did, but it's gone into legend, so we're going to do it anyway. He used to live in the town of Pisa and you may know there's something called the Leaning Tower of Pisa, which is perfect for dropping balls off to see what they do. So I'm going to replace the feather, if I could, by a very small ball. Galileo was interested to know what happens if I drop a very heavy thing and a very light thing from the top of the Leaning Tower of Pizza. Now I'm going to need an assistant to help me with this, but we spotted earlier that somebody's come, actually in a Galileo t-shirt. So I think he deserves to be our assistant. If I could have Galileo's apprentice to come up here, great. Here, if you turn around, he's even wearing the number 17, so this is my man. OK, so if you'd like to come over here. Great, so what's your name, sir?

Tom Tom.

Marcus Tom, what you're going to have to do is see which of the balls is going to arrive at the bottom first. You need to lie down on the floor, do all the mucky work. So if you can lie down on the floor. Great, now I'm going to drop the balls here, so if you can lie that way. Like this, I'll show you, you've got to go like this; I've got to get down. Now I want you to watch and see which ball hits the ground first, and I'm going to go to the top of the Leaning Tower of Pisa. That's what Galileo did; he took his two balls – one very heavy, one very light – to the top of the Leaning Tower, whoa. OK, so Tom, what I want you to do, is to tell me which of these balls is going to hit the ground first. OK, so move your hands away so we don't, great. And if you can count me down: three, two, one. OK, which one hit first?

Tom Same time.

Marcus Same time. OK. Shall we have an action replay, a photo-finish I think we need. Let's see which one did hit the ground first. Here we come. Pretty simultaneous. So let's give our assistant a great round of applause for helping me to repeat my experiment. Thank you.

Galileo realised that the weight of the objects didn't matter at all, what was slowing the feather down, of course, was the air resistance. In fact if I sucked all the air out of our lecture theatre and repeated the experiment with a ball and a feather, both would hit the ground at the same time. So Aristotle was wrong. Galileo started to set about studying how balls fell. How fast does it take for the balls to hit the ground when they're dropping down from the Leaning Tower of Pisa? Well, we're now going to do an experiment which Galileo certainly did do.

Now the thing about the Leaning Tower of Pisa is, if you're dropping balls off the top, they go so quickly it's hard to measure anything. We've got a camera, which we had, so we could slow it down. But Galileo didn't have a camera in his time, so what he did, in order to slow things down, was to use a slope. He rolled balls down a slope to actually investigate what was happening with these balls dropping. So now I need another assistant, another apprentice, who would like to come and help me. Yes, would you like to come up and be a Galileo apprentice for me? Great. So if you want to come here, right and what's your name?

Scarlet Scarlet.

Marcus Scarlet, wonderful name. OK so, what we're going to do is to roll this ball down the slope here, and as you see there are photo sensors at certain positions along the slopes. We've got one here, a little bit further down, a greater distance; and one right down here. When they go passed the photo sensors they make a noise and a little light lights up. So here we go, it goes, like that. What I want you to notice, and the rest of the audience as well, is to notice what happens when it goes past these photo sensors. When does it go past them? So you put your finger there, and we're going to let gravity pull it down. Everyone quiet, and we'll dim the lights so we can see a little bit of the light, so if you let it go. (BEEP, BEEP, BEEP, BEEP.) We'll do it one more time. Hopefully you're noticing something about when it's passing the photo sensors. OK, if you let it go. (BEEP, BEEP, BEEP, BEEP.) So what have you noticed about when it passes the photo sensors?

Scarlet The timing is exactly the same.

Marcus It's exactly the same, it's incredibly regular. So let's thank Galileo's assistant for noticing what Galileo did. That's absolutely right, the distances are much bigger as it goes down, but the time is exactly the same. That's what Galileo noticed, but he noticed a little bit more, in fact. After 1 second it had travelled 1 unit of distance, after 2 seconds the ball had travelled 4 units, after 3 seconds 9 units, and after 4 seconds 16 units. So Galileo noticed the pattern and realised distance travelled. He could now predict it was proportional to the square of the time the ball had been travelling. So in mathematical equations… here's a mathematical equation for you. If you want to predict the future, where the ball is going to be that's D, where it travels. You need to know the square of the time travelled and that would give you some sense of where the ball will be. The half G is about gravity, so you also need to factor that in as well. This is one of the first examples of a mathematical equation being used to describe nature. Using mathematics is a fantastic tool that has revolutionised the way we understand the world. Before this time, people used sort of everyday language, but everyday language is very vague, it doesn't help you to predict things. But now, with the language of mathematics, you can not only describe nature very precisely but we can predict the future.

Now one of the professions which Galileo's equations are used in, day-in day-out, is actually the profession of football. For example, every time Wayne Rooney has got to predict where to go into the box in order to get a cross in from David Beckham, he's actually solving loads of equations in his head. You didn't realise that Wayne Rooney was good at maths, but he is. He's solving these equations in order to be able to understand exactly where to volley the ball into the back of a net. And David Beckham as well, he's also quite good as maths. So I'm going to need two good mathematicians, footballers, a Wayne Rooney. So let's have a Wayne Rooney. Here's my Rooney shirt, he's going to be doing some solving equations. So, do we have any Man United supporters over here? Well they've all put their hands down now. OK, do you want to be Wayne Rooney? There you are, you're Wayne Rooney. OK, you're going to stand over here. This is where the box is going to be, and you're going to try and solve some equations to predict where the ball is going to go. So how about a David Beckham here? Do you want to come out? You can be a David Beckham, why not? Here we go. So David Beckham is going to take a free kick and what Wayne Rooney has got to do, is solve the equations to find out where the ball is going to go. So there you are.

Now an equation is a little bit like a recipe, you have certain ingredients that you need to put into the equation. Then you have the equation, which is sort of the recipe. Mix them together and then the outcome is the answer that you're trying to get. So that's what Rooney's going to try and calculate. I've got an equation machine here. Now this is a little bit more of a sophisticated equation than this one of Galileo's because, actually, now we're doing a little bit of a… Beckham is actually kicking the ball. So it's adding a little bit extra in. Not just letting gravity drop, so we've got a new equation which is going to describe this trajectory. OK, so the new equation, if we can have it up, involved three ingredients to mix up. It looks quite complicated, but all you need to know is the angles. So we need to measure first of all the angle which you're going to shoot the ball out, OK? So let's find out the name of our Beckham, what's your name?

Duncan Duncan.

Marcus Duncan. Can we measure the angle? Let's see, we should have something here to measure the angle. OK, great, that's nice, rough and ready. I do most of my mathematics on the back of envelopes, actually. Here you go; I'm going to put this in here. So do you want to come down and help me measure what the angle is going to be? That's about the right angle somebody's drawn it on pretty accurately, so I'm going to put that in. That's our first ingredient, so if you want to come and put that inside our equation machine. So first thing he needs is the angle. Great pop it in. Now the next thing we need is to measure the speed, which you're going to kick it at. So let's pop the speed inside there. Right. Now the last thing is actually gravity. If you see G up here, it's gravity. If we've got an away match on the moon then actually gravity is very different, and the balls do very different things. So you've actually got to put in; Newton discovered gravity – another myth with the apple falling on his head. So if you'd like to pop gravity inside there. That's our three ingredients, but now we need to know the recipe. Here's our recipe, our equation, if you hold one side of that, OK? That's our equation which Wayne Rooney is going to use, so we'll help him by bundling that in. So you can go back, ready to take you're free kick, but Wayne Rooney. Let's find out who Wayne Rooney is. So what's your name, Wayne?

Hannah Hannah.

Marcus OK, let's come over. What you're going to do is now use our equation and our ingredients to actually calculate a place where you need to stand. So this, our equation machine, it needs to click three times. I think it's the other way, you're undoing your calculation, two, three, great, should be ready. Let's open it up, this little door, and here's our answer. This is the distance Wayne Rooney needs to stand away from the free kick. So do you want to take one end? OK, we've got a position; we've got a target for you to put your foot on, so let's put it at the base of this free kick kicker. OK, if you could put the centre of the target at the end of that piece of string. Yes, move the target; yes, you can move the target to where that's taut. That's great. Now you need to put your foot there because that's where you're going to volley the ball. Are you left footed? You're both footed, gosh! Well yes, you can play for Chelsea Ladies Team. OK, so are you ready? Wayne Rooney's ready, and David Beckham, I've got a whistle here to count us down, we have to wait until the referee blows his whistle. So let's count down the free kick, three, two, one. (WHISTLE.) Pretty close, pretty close. OK, well there are a few more variables which might have affected that one, but let's give our volunteers a big round of applause, thank you. You can go back to your seats. You can keep your shirt on if you want. Well, we've seen how maths enables us to predict where the ball will travel, and I'm going to come back to balls at the end when we go outside and we go try some things with Nicky where we add some spin to the ball as well. Then we'll see how things become even more unpredictable.

Now, you would think from all that I've been saying that predicting the future would always be easy, but that's not always the case. A simple pendulum is actually very predictable and it's why we use it to keep the time. But I haven't got a simple pendulum over here; I've got something called a double pendulum. It belongs to a big friend of mine, Chris Budd at the University at Bath. I want you, during the break, to predict whether when I release this pendulum whether the last rotation of the lower piece will go anti-clockwise or clockwise through the upper piece. Come back after the break to find out.

Part 2

Marcus Welcome back to The Number Mysteries and our quest to predict the future. Before the break I asked you to predict the behaviour of this double pendulum. I wanted to know whether you thought the last rotation would be anti-clockwise or clockwise.

Now some things are very simple to predict, the future. But look at this one, this is much harder to predict exactly. Even when it's going to stop going through the top, seems to come quite stable then it goes round, seems to be going anti-clockwise, anti-clockwise, so you think it's going to end anti-clockwise. Oh no, it's decided to go clockwise. Has it stopped, oh no, no, it keeps going. So you think how an earth could you predict this? It seems incredibly erratic behaviour. I've just made it a little bit more complicated, put two things on, and has it stopped there, do you think that's the last swing of it? So, did anyone notice whether it anti-clockwise or clockwise through last? Yes, what did it go? Anti-clockwise through, but this is so erratic. In fact sometimes it's so surprising it starts kicking again, but I think that is its last go. So, it's incredibly difficult to predict this. This is so erratic that we call it chaotic.

In fact, chaos theory was discovered over a hundred years ago by a French mathematician called Henri Poincaré. He was interested in understanding whether the solar system was stable or not. No one knew whether the planets, as they went on, they might be very stable, but suddenly they might fly off into outer space. Was the thing stable or not? Well it's pretty crucial stuff to know whether the planet we're living on is actually going to be stable and not fly off, so Poincaré began to investigate. Now, mathematically, two bodies are very simple. If I take the earth and the sun, that is very easy to do. It's very periodic; just repeats itself again and again and again, when I do that. But if you add just one more body, a third body, for example, the sun, the earth and the moon, things become much more interesting. I'd like a volunteer to try and explore what happens in the universe when we've got three things in it. Let's have you up. OK, why don't you come here? So this is our little Poincaré, we're going to go over here not from this universe, to our little universe over here. OK, so what's your name, sir?

Harry Harry.

Marcus Harry is going to explore a little universe I've got on the floor here. OK, you kneel down. You can have a look at it. Don't pitch yourself in, gravity is sucking him in I think. Why don't you come round here a little bit, so you're next to me here? What I've got here are three magnets, Harry. They're different colours: we've got a red magnet, a green magnet and a blue magnet. And the magnets attract the pendulum, so this is a bit like the planets and this is like some sort of asteroid which is going to fly around. So, now, I'm going to have some paints coming out of here in a minute and what I want you to do is predict if I let it off from here, which one of the three colours you think it's going to land on. Which one will it stabilise on, do you want to make a guess, if I launch it here?

Harry Blue.

Marcus Blue, OK, you're going to go for blue. So Tim, if you want to remove the paint and we'll get Harry, once the paint's removed, put your hand on it. OK, let it go. So this is now going to map out a little trajectory for us, this is what our asteroid's doing. OK, there's seems to be, whoops there's green and red. I think it likes your blue, but it's a completely wild path where it's going. Is that blue? Seems to be liking blue, oh no, it's gone green, blue, blue. Oh fantastic, here's somebody who can predict the future. OK, Tim if you want to take that one off? Absolutely amazing. Well we've got a budding mathematician in Harry here. Though what I want you to do now, and this is what Poincaré discovered, is that I want you to try and repeat what you just did. So Tim is going to put some more colour on, a different colour, and we're going to see, we know where the path started, we know what direction it went in. Let's see if you can repeat exactly the same path in this new colour. So Tim, if you could give me my new colour, what colour have we got now? Black, OK, let's see if you can, Tim can you help us take the thing off, I don't like getting my hands messy I'm a mathematician. So see Harry if you can predict, get exactly the same path, so we know where it's starting, let it off. That's pretty good, yes. It doesn't look like the same path at all, but will it end up in the same place? Let's see, it started around here, green no, red, red, how amazing, it's started of in nearly exactly the same place, but the path was completely different. Tim, do you want to take that away? It ended up in a completely different magnet, and this is what Poincaré discovered. With the solar system, this is a bit like a little solar system, make a small change in the solar system and things can behave completely differently and go in a different direction. Well, let's give Harry a big round of applause.

So we live in a clockwork universe described by Newton's laws of motion, but what Poincaré discovered is that if you change the initial conditions, just slightly, then the solar system can fly off in a completely different direction. Now Poincaré couldn't sort out whether our solar system is stable or not. Now a friend of mine in Oxford, David Atchison, has done, on his computer, a few simulations of what our universe might be. So it might be a very regular path, like the one up here. This is the one we're hoping we're living in. We'll just keep repeating our path every year,

year-in year-out. But what if we change this, very slightly, the initial condition of one of these three planets then the solar system might do this. Fine for a few years and then things starts to break down, there goes the earth, oh it's come back again, the moon's gone there; the sun's gone there. No, you don't want to live in that sort of universe. He did another simulation which again it looked as though it started very nicely, but after a while, this is another universe that you do not want to live in. Starts off, little knot, elipses but then oh there's goes the moon and the sun out there, OK. Well, just to reassure themselves, scientists have actually run on super computers what is going to happen to the solar system. They've run it backwards and forwards and they've been able to answer what Poincaré was trying to solve, that, at least for the next few billion years, it looks as though the universe will be stable.

Computers have actually helped us to understand a little bit more about these equations of chaos. In fact, I've got over here, a computer print out which helps me to predict what was going to happen in this magnetic pendulum. So I have three magnets down here. So the computer calculated what will happen to the pendulum. So if I'm over a red spot when I release the pendulum from there, it means the magnet will end up at the red magnet, sorry pendulum will end up at the red magnet. But if I release it over a blue spot, then that says it will up with a blue magnet. So here the regions are very predictable. I can move it around a little bit and it will still end up at a red magnet, same here with the blue. You see where Harry set off his pendulum was right over here. And you might be over a little blue spot and go here, but then you shift it very slightly you and you're over a red or a green and it can go somewhere very differently. So, it's important to stress that the equations here, if I started in exactly the same position, it will go to exactly the same magnet. The point about chaos theory is that if I change it very slightly then it can veer off in a completely different direction. Now this chaos makes it very difficult to predict the future. And chaos is everywhere. The weather, for example, is chaotic. It's controlled by very similar equations to the ones that are controlling this pendulum. So, for example, there are regions in the weather, say if I'm here, well this is quite predictable. I change the weather a little bit; this might be a desert or something where it's very hot, if I change it a little bit, it's still going to be hot the next day. And this might be the Antarctic, or something where it's cold, change the conditions a little bit, it's still going to be cold. But we're kind of living, really, somewhere over here, where the weather is incredibly unpredictable. And that's why weather forecasts, you know, three-day weather forecast is about the most we can do. So, for example, you might start off over a blue and predict that it's going to be cold tomorrow, but you move these conditions very slightly, just might be a small decimal place, it might move to the red and suddenly be hot tomorrow.

Now, you might have heard about this thing called the butterfly effect, which says that if a butterfly flaps its wings in Brazil that might be just the small change to shift us from having a nice day tomorrow to having a tornado in Kensal Rise. OK. Well, after the break what I'm going to show you is that population growth is also dependent on the equations of chaos. But first, here's a little problem for you to think about during the break. Which of these three animals throws themselves over a cliff, every four years? Is it a) muskrat, or b) a vole, or c) a lemming? Come back after the break to find out.

Part 3

Marcus Welcome back to The Number Mysteries and our quest to predict the future. Now you might think from what I've just said, that nature divides into problems that are very simple, like a single pendulum, and those that are chaotic, like the weather, but that's not always the case. Sometimes something can start out simple and predictable, and you change it just a little bit and it can go very chaotic. Now, before the break, I asked you which of these animals throws themselves off a cliff, every four years. So does anybody think a) it's a muskrat? So who's voting for muskrats? Hmm, a little bit over here. OK, yeah, what about b) a vole? Who thinks it's a vole? Hmm very few there. What about c) lemmings, loads of lemmings. Gosh are you all following each other or something, maybe over the cliff? OK, well to find out whether it is a lemming I think we should actually find out what a lemming really looks like don't you? So we've got some lemmings to show you in the lecture theatre. Here are our two little lemmings, they are actually very small. Let's bring them down here so our camera can see them. OK, so here are our cute little lemmings. They're actually little rodents, they all from the same family as the vole and the muskrat, but these little lemmings are from the Arctic there. They're really cute. Now, as you can see scientists noticed that every four years the population of, did he bite? Right, the population of these lemmings suddenly plummeted every four years. So people tried to come up with a theory, why every four years are we seeing none of these little lemmings around? Somebody came up with this theory that they were throwing themselves off this cliff, a mass suicide pact, every four years. So it sort of became this myth, and actually here's some footage which helped to reinforce this myth, about the lemmings. So here are the lemmings, sort of coming towards the cliff and then suddenly they decide to fling themselves over the side. I know. But, actually, the people who made this film were so desperate to prove their theory that I'm afraid it was actually the crew who were herding the lemmings over the side. And so, but I assure you that our crew have been treated these lemmings with a lot of respect, they've been petting them away, in fact, we've got a little sort of auction of, who wants to take a lemming home. Anyway, I want to say goodbye to our lemmings in a little bit, but let's keep them here. What is the explanation, if it isn't evil cameramen or a mass suicide pact, what is it that's explaining the sudden plummet in their population? It is in fact, mathematics. So let's take our lemmings away, our little live lemmings. I want two people to play some lemmings. So can I ask you, would you like to come up and be one of my lemmings? And I need, OK, why don't you come up and be a second lemming. OK, so here are our two little lemmings, aren't they cute? OK, so let's find out what our lemmings are called? What's your name?

Abi Abi.

Marcus Abi. And what's your name?

Josh Josh.

Marcus Josh and Abi are our two little lemmings, OK. So there's a little mathematical formula which tells us how the lemming population changes from one season to another. So what happens to these two little lemmings is they become four lemmings. We're going to double the lemmings each time, so I need now two more lemmings to join the lemmings in the next season. OK, let's have you over there, yes, and let's have you as well. Now, so we've got our four lemmings, let's put you in a line here. We've got four lemmings now, but there's not enough food for all the four lemmings, unfortunately, so the little mathematical formula is going to tell me how many of these lemmings are not going to survive the season. OK, so the mathematical formula is, I'm going to have to take the two I started with, multiply by the four I've got now. So 2 x 4 is 8, and divide by 10. Here's the formula up here, and I've actually got my equation machine, so 8 over 10 is about 1 lemming. Even I can predict that 1 lemming is not going to survive. So we're going to play a game of musical lemmings, in order to decide who is going to get through. So here's 3 chairs, 4 lemmings, cue the lemming music. OK, round the chairs you go, that's exactly. Oh I like this lemming, when the music stops grab your chairs. Ooh, aww, gosh this one went over cliff. I heard this one, so I'm afraid this poor little lemming has not survived the season, so let's send him back to his seat. I know, I'm sorry, but here are our three survivors, up you come. Let's have you in a line. Next season they double up again, so I need three more lemmings to join these one. OK, so let's have you, OK, and why don't you come up, yes, and let's have you. OK. Ah, don't worry, there's another round yet. So now we've got, oh this ones having a lot of trouble coming out but we'll wait for her, don't worry. So we've got 1, 2, 3, 4, 5, 6 lemmings, whilst I do my calculation you can come down. So now I have my 6 lemmings, but how many are going to survive this season? Well I've got to take 3 x 6, divide it by 10. I've got my equation machine here. Unfortunately there's not enough food for everything so two are going to die, but four will survive. So now we need 4 chairs inside here. OK, survival of the fittest this game. Let's go, so here we go, cue lemming music. Come on cheer them on, I'm a lemming. OK, who's going to survive? I like the lemming dance. Oops, ooh there we go, our two lemmings, I'm afraid who didn't survive the season. Yeah these two are doing very well, OK, up you come, my four lemmings who've survived through to the next season. OK, so we need to double up again. Now I need 4 lemmings. OK, well let's have you there, the little one there, and let's have you. Come up OK. And why don't you come up. OK, so how many have I got? I need another female lemming, I think. So let's have another female. Yes, why don't you come up? OK, let's line our lemmings up, have I got enough now, 1, 2, 3, 4, 5, 6, 7, 8, and now its fierce competition for food. Who's going to survive now? We need to do our calculation: it's 4 x 8 divided by 10. OK, I'm afraid 3 of you aren't going to survive this time, so 5 will. So 5 have got to fight it out now. Cue the lemming music. OK, who's going to survive this one, fierce competition for food? I bit of hogging the seats there, I know. Is it going to go, oh my God, there are 3, 4 lemmings, well you did pretty well. Let's give them a cheer, OK. So now, something rather interesting happens with this equation. I've got my 5 lemmings here, next season they double up to 10, 5 x 10 divided by 10 is 5. So every season from now on there will always be 5 lemmings. This mathematical formula has predicted that the lemmings will stabilise. So I've got a little graph here to show you the stabilising of this lemming population. So actually however, wherever you started, if I started with 7, it would have come down to 5. So with this equation, everything's very predictable, and it stabilises at 5 lemmings. Let's give our 5 lemmings, the surviving lemmings, a big round of applause, great.

Now look what happens if I change the equation a little bit more. This time, rather than doubling the lemming population each season, I'm going to triple it each season. So if I triple the lemming population, use the same formula we find that the lemming population does something very strange. It actually ping-pongs between two different values. So here we have it: sometimes there are lots of lemmings, sometimes there are none; lots, none. This is the explanation for the story, the myth of the lemmings. Because if you tweak it a little bit more, you get four different values which it ping-pongs around, and one's very small. So every fourth year it suddenly plummets then it comes back up again. OK, but the amazing thing is that if I tweak this model a little bit more, instead of tripling the population of lemmings, what happens if you quadruple it? Then the population goes completely chaotic. It's almost impossible to predict what's going to happen. So here's the population of lemmings when they quadruple every season and we use the same formula. Sometimes it plummets down and then it flip-flops around, and it's really hard to understand it. In this model the behaviour of the lemmings looks totally irregular and unpredictable. Yet there is a very simple equation, hiding behind here, which is predicting these lemming numbers, and this is chaos at work again. Now this is really important, for example, for ecologists, who want to understand what's happening in species which might be endangered. It could be a change in environment which is causing it, or it could be just simply the result of chaotic equations.

OK, we've seen from the lemmings that things may start out simple and predictable, but you tweak it a little bit and suddenly it becomes chaotic. Now this switch of behaviour is actually responsible for those dramatic free kicks that you might have seen in football. So we've going to go back outside now, to Nicky de le Salle, who I hope is not too cold out there. Hi Nicky, how are you doing?

Nicky Not too bad, thanks, nice and warm.

Marcus Nice and warm? OK, you're warmed up. OK we'll get you back inside in a moment, but we want to see a few dramatic free kicks. I can see a goal in the background there, so we want to try and explore a little bit of the mathematics if you put a bit of spin on the ball, what happens to it. OK, could you give us a free kick, please Nicky.

Nicky OK.

Marcus OK, so I can see the goal there. Right, we saw what happened when we just kicked the ball straight, it goes straight. But now we're putting some spin on it, here goes. OK Nicky. Ooh. I'm cheering because I'm an Arsenal supporter, hurray she missed the goal.

Nicky 1-0.

Marcus OK, hold on, I certainly saw the bend there, but you didn't get it in the goal. That's alright we want to do the bends, we're not interested, so now can you put more spin on it, and get an ever-bigger bend?

Nicky I'll try.

Marcus Ah, OK, it's one-all Nicky alright. So maybe we'll sign you up for our team, well I play for team in the Hackney Marshes. Do you want to come and join us for a bit?

Nicky Think I'll stay around, to be honest.

Marcus Yes, probably wise. OK, well let's give Nicky a big round of applause for helping us spin the ball. Thank you Nicky, good luck with your season. Now, what I want to do is show a little bit of the mathematics about why some of those really dramatic free kicks, where suddenly the ball is going and then very late on, the thing bends. And the answer is turbulence. We've got a little turbulence to show you, so Andy if you'd like to bring on. This is some liquid nitrogen, it's going to fall off the stick and you'll see what's happening, sort of smoke. Is it ready to go?

Andy Ready to go.

Marcus If you look at this, at the very top, it's very smooth, ooh I shouldn't go too, it's very smooth and regular, but as it hits the bottom it become incredibly chaotic and hard to predict. This is actually, these two different behaviours, is the key to understanding some of these really dramatic free kicks, like the one Beckham takes. OK, let's take this away, and I want to explain now why those two different sorts of behaviour are responsible for dramatic things that happen to a football. So, what happens is that the football starts spinning, and Nicky there she starts by hitting it incredibly hard and at very high speed, the turbulence behind the ball is actually like the bottom there, it's very chaotic, and the chaotic turbulence behind the ball actually doesn't put very much drag on it. The air sort of shoots by very quickly, so the ball goes very quickly until, at some point, the behaviour changes and it goes from chaotic to a very regular flow behind there. And that regular flow clings to the side of the ball and acts like a big brake. So it's like a brake slamming on this ball, it suddenly slows down and then the spinning suddenly comes into effect. Then we have this dramatic sudden bend at the last minute. So that's the key to the late bend in the football, is this swap from chaotic behaviour to very regular behaviour. Now, the mathematics behind the turbulence is actually not completely understood. In each lecture this week I've told you about an open problem that we can't solve, for which there is a million dollars. Now the million dollar problem for this lecture is actually understanding the equations which control the turbulence behind that football. It's very important because something like an aeroplane wing, it's really important to understand what's happening behind the aeroplane wing. So there are certain equations, called the Navier-Stokes equations. If you can solve those equations, the million dollars will be yours.

For me, frankly, the money is not what motivates me. It's the buzz of cracking one of those great unsolved problems, that's been around for years, which gives me my buzz. Frankly, I would pay somebody a million dollars to really understand one of those five problems. I first learnt about that buzz of doing mathematics, when I was sitting in the audience, like you, in 1978 when Christopher Zeeman gave the Christmas Lectures. It was the first time that the Christmas Lectures had ever been given on mathematics, since they started in 1825. I was so amazed at how exciting and magical this world of mathematics is, that I decided that I wanted to be a mathematician like Christopher Zeeman. So I hope that these lectures, over the past few days, have shown you a little bit of my magical world. Thank you.

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