THE NUM8ER MY5TERIES –
Presented by Professor Marcus du Sautoy

LECTURE 3: THE SECRET OF THE WINNING STREAK

Broadcast on: 27th December 2006

Part 1

Marcus Hello and welcome to The Number Mysteries. Rock, paper, scissors. It's a pretty simple game: rock beats scissors, scissors beats paper, paper beats rock. But we'll see in a little bit why mathematics can give you a bit of an edge in this competition. Now we've been running a little competition here to find a Christmas Lectures champion, and it's down to these two over here. So we're going to do one final round to find our Christmas Lecture champion. Are you ready? We're going to count you down, have your fist there. Right, you're going to go when I go three, two, one, and then draw. OK? So it's three, two, one, draw. Whoa, another round. Three, two, one, draw. Ah ha, and here's our champion whose won with paper, and so what's your name?

Will Will.

Marcus Will is going to come a bit later, so well done Will, a large round of applause. You can all go back to your seats now, and Will, I'll get you down in a little bit later on. He's our Christmas Lecture champion, but we're going to get you to take on the World Champion in rock, paper, scissors, a little later on, and we're going to prime him a little mathematical strategy which we hope might get him the World Championship.

In this lecture I'm going to reveal some of the mathematical secrets behind some of the games that we play. Now we're also going to play a game for which you can win a million dollars, if you can find some winning strategies. A million dollars. But we're going to start with a game that a lot of people play each week, to try and become a millionaire. And that's the National Lottery. Of course, you've got to wait until you're 16 before you can legally play the lottery, so we're going to play our own little National Lottery inside here, in order to try and find out some of the mathematics of winning the lottery. I asked you, before you all came in, to fill out your lottery form and you should choose 6 numbers out of 49, and if the 6 numbers that come out of here are the same as the 6 ones, then you win the lottery. But what is the chance of you winning the lottery tonight? Well, mathematics, as some of you know, I think. But mathematics tells us there's about a 1 in 14 million chance that you will get the right numbers when these 6 come out. So it's more likely that I'm actually going to die during the course of this lecture than one of you is going to win the National Lottery, but I don't want to try that one out, if possible. So, now I hope you've all finished and I'm going to really trust you to mark your own papers so don't cheat please, because the maths will be able to find you out, if somebody really does get 6. Or there is a chance you could get 6 then, anyway. But I need a volunteer whose going to chose my 6 numbers here, so who would like to come and choose some 6 months? Yeah, would you like to come, great. Ok, what you need to do, you can come over here, here's our lottery machine, a very, very high-tech one. So, what's your name?

Megan Megan.

Marcus Megan if you would like to switch on the lottery balls and they're going to come shooting up randomly and we're going to get 6 balls out here. That's right, switch it on, and here we go, and we've got the first ball already, so Megan if you'd like to take the first ball. There we go, it is the number ...

Megan 47.

Marcus 47. OK, let's put that up here, has anyone got 47? Right, and our next ball please, Megan.

Megan 43.

Marcus 43, OK. So we're all the big balls. 43, prime numbers good and ...

Megan 17.

Marcus 17, it's rigged to give me lots of prime number, excellent. That's the one I play in my football team. OK, fourth ball.

Megan 41.

Marcus 41, wow, this is extraordinary, look at that, 41, 43, 47. OK?

Megan 21.

Marcus 21, oh dear, that's broken our pattern. 21, and one more ball please Megan.

Megan 22

Marcus 22, that's fantastic. OK, switch off the machine, 22, so we've got 7. No we've got 17, 21, 22, 41, 43 and 47. Let's give Megan a big round of applause for choosing the balls. You have to check your numbers now to see whether, maybe she actually won, maybe she's rigged the balls, I don't know. I'm going to get you all to stand up now, with your lottery tickets, please. If you'd like to all stand up. Now, I'm going to use a bit of mathematics to make a prediction that about half of you haven't got any of these numbers right. So I want you to sit down now, if you didn't get any numbers right. Oooh, what do you think? I think that's about half, I think that's not bad. Right, two numbers, who's got two or more numbers? I will predict, using mathematics that I think that about a quarter of you will still be standing. So please sit down if you only got one number right. Ooh actually, I don't think you look too lucky, I think that's a little less than a quarter actually. OK, what about three balls? OK, so I want you to sit down if you've got two or less balls. Hmmm, we're really thinning down now. Gosh, there's only you left. Oh there's one over here, that's one, two. Now that's interesting, because I predict, actually that there should be about 7 or 8 of you, because there's about 400 people in the audience, and there's a 1 in 57 chance that you should have 3 numbers right. So 1 in 57 chance should leave you with about… so we're actually doing not terribly well here. OK, now let's see how many of you got? 4. We've thinned you all out, yeah, that's not surprising, because now it's a 1 in 1,000 chance that you would get 4 numbers right. And actually, you'd have to buy a lottery ticket probably every week for about 20 years before you get a ticket, with 4 numbers right, so that's pretty rare. OK, 5 numbers. We've got nobody with 5 numbers – also not surprising because you'd have to a buy a lottery ticket every week for about a thousand years for that to happen. And the National Lottery, to win 6 numbers, well that's incredibly slim. As I said, it's 1 in a 14 million you'll get all 6 of these numbers right. In fact, if the first homo sapiens, his first thought had been to go down to the newsagents and start buying lottery tickets every week up until now, he might have won the lottery about once. That's pretty bad odds actually.

Unfortunately, I don't have some special mathematical formula to try and find these numbers. If I did, I assure you, I wouldn't be talking to you now, I'd be relaxing on some tropical island enjoying myself. But I do have a mathematical strategy to help maximise your winnings if, by any luck, you do actually get all 6 numbers right. Because what you don't want to happen, is what happened in the ninth week of the National Lottery when 133 people got all of the 6 numbers right. They must have been ecstatic, thought they'd won the jackpot. They phoned up, and they found they had to share the £16 million prize with 132 other people, and so they only got about £100,000. Now I'd be happy with £100,000, but when you think you've won the lottery and you only get a £100,000, that's pretty bad. Now here are the numbers that they chose. OK. The numbers they chose are 7, 17, 23, 32, 38, 42 and 48. Now you see quite a few prime numbers in there, like our ones. But there's something rather special about this, they're very nicely laid out. There's sort of a beautiful gap between there, and this is what people tend to do when they try and choose random lottery tickets. They like to space them out in a nice sort of pattern. That's the trouble: people choose nice patterns like that, and in fact, quite a lot of random numbers, clumped together. In fact, if we look at our random numbers, we've got 21 and 22 together. Now I want you to stand up if you chose lottery ticket numbers with two consecutive numbers, like 8 and 9, or 36 and 37, or like our 21 and 22. OK, quite a few of you have clumped your numbers together, but the mathematics says that half of you should have chosen numbers with two together. Half of those numbers that come out of here, six numbers will have two that are next to each other. So it shows that that's not half, we're not very good at choosing random things. OK, let's sit you down again. That's the trouble people are finding it very difficult to choose random things. Everyone tends to space their numbers out, in these nice patterns like we saw there. And so they're going to end up having to share their winnings. So here's a little strategy for you, to maximise your winnings, try and clump your numbers together. Then you'll probably only share it with a few people that stood up.

Now, making things random is also the best mathematical strategy for winning this game we started at: rock, paper, scissors. So I'd like you to give a big round of applause to our Christmas Lectures champion, Will, whose going to come down. If you'd like to come down. Where's Will? Somewhere here, great. Welcome back Will, we'll put you down here. Right, face this way. Now in a minute Will we're going to get you to play the World Champion at rock, paper, scissors. He's going to come in, and actually he lives here in Britain, in North London, his name is Bob 'The Rock' Cooper, so we've re-named you Will 'The Scissors'. You're going to play, Will The Scissors is going to play Bob The Rock. Now Bob The Rock is incredibly good at spotting patterns, right. He's a sort of secret mathematician at heart. So if he can pick up a pattern in what you're doing then he's going to able to beat you. Because if he knows you're going to do rock next, then he'll do paper, to get you. So we've got our own little mathematical strategy. If I could have my strategy, does anyone have my strategy cards? What we're going to do is try and randomise what you're going to do. I've got loads of cards here with our decision on, and all I'm going to do is shuffle them. So it's a very simple mathematical strategy, but we think that this should take the edge off Bob The Rock, who's looking for a pattern, if we cannot get any pattern at all. So let's shuffle them up and I'm going to show you these as we're going along, we should take the edge of it. It will at least give us a 50/50 chance, evens chance, of beating the World Champion. I don't think there are many other sports were I can find a strategy, a 50/50 chance of beating the World Champion. I mean, 100-metre sprint, I don't think so, you know, so I'm hoping this is going to help us beat Bob. Now before we introduce Bob The Rock, the World Champion, let's see him when he was winning, in Toronto a few weeks ago, and see if we can pick up a few tips. So this is Bob The Rock on the left here, in the dark glasses. And there's him winning, there it is, he's done the winning one, scissors, absolutely ecstatic, and here he is with his trophy. I'd like you to give a big welcome, a Christmas Lecture welcome, to Bob The Rock Cooper. Right, let's have you over here.

Bob the Rock Alright.

Marcus Well, with your trophy as well. Well I'm going to take your trophy for a while and we're going to put it over here. Well it's very sporting of you to come in, Bob, and we're going to play. I'll introduce you to Will The Scissors, our Christmas Lecture champion. Now what sort of warm up do you do for the World Championship?

Bob the Rock Well, up to the event, I did a couple of hours a day training, and before that, normally about an hour a day.

Marcus Couple of hours? So a couple of hours of doing this all the time?

Bob the Rock A couple of hours, making sure that I can see what's coming out, as they come out.

Marcus Amazing, ah ha, so you don't want to predict what you're going to do?

Bob the Rock Exactly.

Marcus And do you have a strategy when you played and won the World Championship?

Bob the Rock I do have a strategy, yeah, but I'm not going to tell you.

Marcus Oh I wonder why, yes. Well we have our own little strategy and we're going to do it, I'm going to try it out on you. Will, if you'd like to come here? I'm going to get you close together. So first of all, let's do a little practice run, so we know exactly what we're going to be doing. If you put your… one fist out each, and we're going to do, three, two, one, draw. OK? So let's have a practice round. Three, two, one, draw. Whoo. But that one doesn't count, that was just a practice round, OK? So we're going to do best of six for the World Championship. The first thing you're going to do Will is this, ready? Three, two, one, draw. Hmmm, didn't work that one. Well it's one to Bob The Rock. Wait a minute, this is our secret strategy. Three, two, one, draw. Oooh that's two papers, I think, yes, so that's a draw. Got to be clear with what you're doing. Next strategy. Right, three, two, one, draw. Oooh, two to Bob The Rock. He's pretty strong at this. You can still get it back, you've got three more rounds, alright, you've got the next one. Three, two, one, draw. Whoa, that's Bob The Rock, has retained his championship. Bad luck, our strategy was, it only gave us a 50/50 chance, so don't worry, we will give Bob his trophy back, not bad. But we do have a trophy for you, there you are, don't worry, we're not going to steal it off you. But we do have a trophy for our Christmas Lecture champion, who did very well; and here it is – a little bit smaller than Bob's I'm afraid – but, give Will a big round of applause. Right, thank you Bob. That's very sporting of Bob and obviously his strategy is better than ours, and he's not going to tell us, so.

Now, after the break we're going to play a game, and we're going to see how maths can help you win at Monopoly. Now Monopoly, you might think, just depends on the throw of a dice, but mathematics can help you get a bit of edge in that game too. I hope, anyway. While, during the break, I want you to think about a little question: how many faces do you think the first dice in history had? Do you think it had a) two faces, or b) four faces, or c) six faces? Come back after the break and find out.

Part 2

Marcus Well, before the break I asked you how many faces did the first dice in history have, and the answer is in fact, 4. I've got some of them here. So Mark, if you'd like to come in and have a look at this. They are in fact called knuckle bones and if I throw them. I'll show you one; they have genuinely got four different sides that this knuckle bone can land on. So these were actually ankle bones of sheep and they were used over 5,000 years ago, for games of dice. So if I throw them on the floor here let's see what I've got, well you can see two have landed flat down and two are upright like this. You can see these are pretty biased dice, and they're going to be landing on one side more than any other. So the ancients, they started to try and carve these dice, to make them more symmetrical. They began to start to look more like the shapes that we looked at in yesterday's lecture. And some of the first really symmetrical dice I've got here. This is a game that you can find in the British Library actually. It's called the Game or Ur. In this game they had little tetrahedral dice, so I've got them here – these are like little pyramids – and these have four faces, but they're perfectly symmetrical. Now you can see that some of them have a little white dot on one corner, and there are two white dots and two black dots. And so the way I would score this game, is if I take the four dice and I throw them. OK, so I've got one, two with whites dots and this one had a black dot and a black dot. So this would give me two points, so I'd move my counter on two. This was a game played 5,000 years ago by the ancient Mesopatamiums.

Now actually, of course, today we use a six-faced dice, and one of the best games that I love at Christmas time, with dice, is Monopoly. But how can mathematics help you to win at Monopoly? I want a volunteer to try play a bit of Monopoly to see how mathematics can help you. Let's have you here. OK, would you like to come up? Right, so we've got a great big Monopoly board here, we need to go down to the Go. That's where we start. Now we've got three counters, which is your favourite counter out of these? The iron? Great, right, and what's your name, I need to know who I'm playing?

Ila Ila.

Marcus Ila, OK. So what we've got to do, first of all, I want you to try and think which of these squares on the Monopoly board is the most visited square in Monopoly. Is it maybe Angel Islington? That's about the nearest place to where I live. It could be The Strand, over here, that's where our lecture is, next to free parking. Though I haven't seen much free parking when I've come in, I must say. But, so what do you think is the most visited square on the Monopoly Board?

Ila Whitechapel Road.

Marcus Whitechapel Road? That's an interesting choice, because you think it's near the beginning of Go, and things. Once the game starts going, actually the Go square isn't so important. I'm going to give you a little hint: it isn't Whitechapel Road. It's a nice choice, but what happens when you go on this square. You go to Jail, and so let's take our thing round to Jail, because it actually Jail is the most visited square on the Monopoly Board. So let's come round here. Actually there are many ways you can go to the Jail, if you take a Community Chest card – I always seem to get the ones which send me into Jail – I don't know about you when you play it. I never get the 'Get out of Jail free' card. Also if you throw three doubles in a row, you get sent to Jail, which I think is really unjust, you know, really lucky and suddenly you have to go to Jail. So, in fact, Jail is the most visited square on the Monopoly board. Let's make you 'Just visiting' for a while. Now, OK, so we can't buy Jail, that's not terribly useful, so let's throw the dice and see where you end up to, after you go to Jail. So if you'd like to throw the dice for us. OK, and you got 8. Do you want to move your iron on 8? 1, 2, 3, 4, 5, 6, 7, 8, and Marlborough Street here. Well 8 is actually quite a common throw after you've come from Jail. Here are all the possibilities of the throw of two dice. So down at the bottom here, a very rare one, we've got two ones – there's only one way to get the score 2, that's with one and one. And right up at the top there, to get double six, there's only one way to do that. But in the middle here, so you want to stay next to your thing. In the middle there's actually, look at this, six different ways to get a 7. So actually the most common throw of a dice, with two dice, is actually 7. Well actually, where does 7 get you from Jail? I think it's one back from here. It gets you to Community Chest, but you still can't buy that. But as you discovered, after that, the next most common ones are 6, that's pretty high, and 8. So just as you found, a lot of people when they go to Jail, are going to find they're landing on Marlborough Street or Bow Street. You were first there so you can buy that property. That's your strategy, as soon as you get on that, buy it and stack it with as many hotels as possible. Then everyone who's coming to Jail, quite often, they're going to land around there, with a throw of the dice, and you're going to raking in the cash, and you're going to strip everyone at Monopoly. So there's your strategy for winning at Monopoly. Give her a big round of applause, and we'll send her back. Well, I'm the loser so I'm going to have to clear this thing up. I'm going to get my team to help me clear up. That's the rule; you have to clear up your games, of course, when you've finished with them. But you still need to be lucky to land on orange.

So now I want to show you again where mathematics can guarantee you at least a draw. Who'd like to come and play 15 with me? I want somebody, yeah, why don't you come? Right, come on down here, and I want you stand behind your cake stand here. Now what's your name?

Lottie Lottie.

Marcus OK, Lottie. The game here is I've got loads of chunks of cake behind me. One slice down here and up here I've got a large chunk which consists of 9 slices of cake, but your cake stand, it takes 15 slices. The game is you've got to take some slices, chunks of cake and try and make a complete cake, but you have to do it with exactly three chunks of cake. So you're going to try and take three chunks where the numbers add up to 15, OK. And we're going to take it in turns to play the game. I'm going to try and fill my cake as well. So off you go, you can start in this game, alright? You understand the rules?

Lottie Yes.

Marcus OK. Choose any piece you want. She's going for that big one, first of all, why not, that's good. Try and fill it quickly, so I'm going to go for this one, I'm going for five. Right and I have to watch what she's doing, so she's gone for 4. OK, not sure, probably… I'm going for 2; I think I'm going to stop her, so go for 2. You can take your piece off and try a different one because I've just scuppered you with your strategy there. So those are your pieces, you can keep those pieces and you can choose some more pieces, OK. So you've got some other pieces here. Do you want 1, 3, what do you need? Oh there's lots of people shouting. Don't help her, God it's one against 400, that's not fair. I'm going to take 3 I think, right, well that doesn't work. Has she got, did I manage to get the piece that she needs? OK, your turn, you've got 3 left, what are you going to do. No helping her, shhh, it's not fair. OK, she's looking at mine; that's fair enough, you know. Now I'm going to win now because I've got 7, yes, and I can take that, and 5 plus 3 plus 7 wins me the game. OK, so even against 400 I managed to beat her. So not bad. It's a pretty difficult game because, to start with it's quite easy, you're trying to add up the numbers to 15, but after a while you've got to keep track of so many different things: what I'm doing, what you're doing, different combinations.

Now the reason I won that so quickly, was that I was playing a different game with you. I was actually playing noughts and crosses with you. How come I was playing noughts and crosses? It doesn't look like noughts and crosses at all. Well here's the noughts and crosses board that I was using. Here it says on the screen, it's actually a magic square, some of you may have seen this magic square. All the ways that the lines that you can draw in this thing – rows, columns, diagonals – they're all the ways that you can add up to 15. So if I played noughts and crosses with you, and I got 3 in a row, which is what I got here, 7, 3 and 5. So 7, 3 and 5s the middle column, I won the game. I was actually playing noughts and crosses with you, which made this really easy to play. So, what we're going to do now is I'm going to give you this strategy and you're going to play me again and you will see how much easier it is when it becomes a game of noughts and crosses. So if you could sit back down in your seat for a while, and I'm going to pull you up in a little bit.

Now what I'm going to do is to rearrange this board, so it looks like the magic square up here. And as if by magic, I've got a magic square. So now I'd like my volunteer to come back on again and we're going to play noughts and crosses, but actually our noughts and crosses game is going to be the same as the game we played, of 15. Here are our noughts and crosses. Now, what do you like? Do you like noughts or do you like the crosses? OK, crosses, and you get to go first, like I gave you the chance to go first last time. Now do you know a good strategy for noughts and crosses, what's the best place to go on noughts and crosses, the guarantee? The corner, I actually think the middle is pretty good. OK, you can play corner, you can play corner. Now I think the really bad move, for me, is to go here. I think. Could you force a win from there? Let's see if you can. Right now, if we were playing 15 I would have seen she'd got, the next one is 6, by 5 so I know that she's going for 4, so I'll block that one there. But like in noughts and crosses I can see very quickly that that's what you're going for. Your go now. Now that's a good move, isn't it? Because look now there are two different ways for her to get 15, this line here and this line here, and I can't stop her. So I'm going to go here, try and stop that one. And she wins the game like that! Now isn't that so much simpler than playing 15 and trying to add things up? Let's give her a big round of applause for beating me. Oh and, and we even have a very small cake, I'm afraid for, for the winner, so that's great.

In fact, this is one of the powerful things in mathematics, to try and change a game, or a problem you've got, into something else. And suddenly, from a new perspective, the thing might become much easier. It was quite difficult when we were trying to add all the numbers up to 15, but playing noughts and crosses – anyone can play noughts and crosses. Well, in the next part of the programme, I'm going to show you some games where even mathematicians haven't come up with a winning strategy. But during the break, here's a little puzzle for you. I've got two graphs here, I want you to have a look at them and say which of these graphs you can draw without taking your pen off the paper, and without going over a line twice. Is it a) the picture looking like an envelope or b) the one that looks likes 83 sort of joined together? Come back after the break to find out.

Part 3

Marcus Well, before the break I set you a little challenge. Which of these two shapes can you draw, without taking your pen off the paper and without going over a line twice? Well, this little problem relates actually to a game that residents of Kurningsburg used to play 200 years ago, and we built a little miniature version of Kurningsburg for you here. They didn't play mini golf, what they did was, well Kurningsburg here, has two rivers flowing into the town and we have an island here in the middle, then the rivers met again here and went off to the sea. There were 7 bridges in the town of Kurningsburg and of a Sunday afternoon the residents used to play this game, where they would try and walk around the town, and try and go over each bridge once, and once only. So I need a volunteer who's going to play a resident of Kurningsburg. Who would like to play a resident of Kurningsburg? OK, why don't we have you sir? A very tall resident of Kurningsburg here. Right now the game is to try and get around this town, crossing each bridge once, and once only. You can start in any of these islands, we've got A, B, C and D over here, where do you want to start? And what's your name?

Darren My name's Darren

Marcus Darren. OK, and he's the resident of Kurningsburg, where would you like to start?

Darren D.

Marcus OK, off you go to D and then we'll start, and I'll follow you round. I'll put the flags down. Which bridge would you like to start with? OK, off you go. Right I'll follow him around. Put that down, one down, six to go. We're on Island A now, which way to do you want to go? This is a real gentle stroll, gosh, I usually jog round Kurningsburg. Whoops, don't want land in the sea there, gosh. Right you've got quite a few choices on this one, but the residents of Kurningsburg can't swim, so you want to go back over D?

Darren Yes.

Marcus OK, let's go over D. Great. Well we've got no choice here; we've got to go over this one. OK, he's beginning to sweat here, I think. So, pretty stressful. We've got a few bridges left, three bridges left. Which one do you want to go over, this one or this one? OK, down it goes. Right you've got two bridges left to go over. It looks like, which one are you going to go over? This one up to A? Or this one to C? OK, off you go. Oh dear, looks like we've failed because there's one bridge there we cannot get to it. OK, let's give him a round of applause for at least having a go. Well our volunteer actually discovered what the residents of Kurningsburg found, because whenever they played this game, they put all six of the flags down, there was always one left that they just couldn't put down. They began to wonder maybe there was a combination we haven't tried, so they kept on playing the game again and again. They had to wait until a Swiss mathematician, Leonhard Euler, arrived in the town in 1736 and he managed to sort this problem out for them.

What he did was to change the game into something else. A bit like we did with 15, and change it into noughts and crosses. So what he did, was to change the map of Kurningsburg into a graph, so he condensed all the islands down into these points, we've got four points – A, B, C, D – and he connected the points if there was a bridge joining island A to island B. So for example, there is a bridge here, so I need to connect Island A up to Island B. In fact there are two bridges, so there are two lines that he would draw from these points here. So if I connect it up we get a lovely graph. OK, if we take the lights down, we see actually Kurningsburg changes into the graph that you're all trying to draw during the break. And now this game of trying to get around Kurningsburg crossing each of the bridges once, and once only, is in fact that same game as trying to draw this graph without taking your pen off the paper and not crossing any line twice. OK, let's bring the lights up and see how Euler found a way to solve this problem. What he realised is, that as you're going around this graph, every time you come into a point you have to go out on a new point. So the number of bridges coming in and out of a point has actually got to be even. Well, there are two exceptions, the beginning, that's one bridge coming out, and where you end up, that's one bridge coming in. So there must be a maximum of two points where there are an odd number of bridges coming out. OK, let's take the lights down again and count how many bridges there are coming out of these graphs. OK out of this point I've got one, two, three, well that's already one odd point. Let's move to this one B, I've got one, two, three, four, five, I've already got two places with an odd number of bridges coming out. And if we move to C, we've got one, two, three, three places. So actually it's impossible, Euler showed, to get around the bridges with crossing each of them once and once only. In fact, if we get over to D here, stride over to D; we've also got this one with an odd number. So one, two, three, here coming out. All of them have got an odd number of bridges coming out, so it's impossible to get around Kurningsburg. So don't worry our volunteer who tried to get round it, nobody can get around it.

But Euler did something even better. Euler in fact showed that if you do have a graph, where there are only two places with an odd number of bridges coming out, that actually you will find a way to get around that graph. So, for example, the graphs I showed you during the break, that little envelope there, if you count up the points; there is in fact just two places, the bottom left and the bottom right hand corner. And here's a way around that graph. In fact, Euler gives you a strategy; you start at one of the odd number places, and finish at one of the odd number places. So there, there's a way to get around the graph with the picture of the little envelope by using a bit of mathematics.

Now this problem of the bridges of Kurningsburg is an example of a problem mathematicians love. Because, rather than trying out all of the different combinations and getting exhausted, there's a clever and lazy way to show that it's impossible. Or to show on other graphs that it is possible. But there are other problems for which mathematicians haven't come up with any clever strategy to solve. And the last game we're going to play tonight is in fact a game that didn't take place in the eighteenth-century town of Kurningsburg, but we've got to move up to the North Pole. We find ourselves in Santa's grotto, and it's Christmas Eve, and Santa is trying to decide which elves he's going to take with him on a big trip. So who's going to be lucky? He's got a game to try and decide which of the two elves he's going to take on the big Christmas Eve trip. OK, so I'm going to need four volunteers to play my game. Let's have, I'm going to have, you, OK, come on up, and let's have you, right, this is one team. They're the red team of elves and you've all got to support the red team, OK, so here are you're, you're red, go red, go red. And on this side I'm going to have, you can come up sir, and you'd like to come up. Right you're the green team. There are your elf hats. If the green team would like to come here, right you're all supporting green, OK, go green. So let's find out, let's find out who our elf teams are, so what's your name?

Amy Amy.

Marcus Amy and ?

Callum Callum.

Marcus Amy and Callum are the red team. Let's hear it for the reds. OK, if you'd like to come in here, good right and who's for the green team, we've got?

Amy Amy.

Marcus Amy and ?

Ollie Ollie.

Marcus Amy and Ollie. Let's hear it for the green team. Now, Father Christmas has agreed to take the elf team, who can stack this sleigh and fill it as much as possible with the presents that we've got here. The length of sleigh is 150cm here, and each of the boxes has a number saying its length on, so here we've got a box which is 42cm. Now you have to stack them in a line like this, so the numbers are on the top and they're along the bottom and they're along here. And I want the numbers to try and add up, if you can, to get them to add up to 150, but Santa's going to take the team that gets nearest 150. Be warned, you cannot go over 150 because poor Donner and Blitzen here, they will not be able to take off if it goes beyond that. OK? We're going to give you 30 seconds to try and pack this sleigh, as far to the bottom as possible. OK? So we're going to count down, three, two, one (WHISTLE). Let's hear it for the red team, come on, go red, go red, come on. They've gone for a big parcel, and here we have the green team, go green. Alright. How many seconds, I've got it. 10, 9, 8, 7, 6, 5, 4, 3, 2 (WHISTLE). Let's stop. OK, there's your decision. Right we have to find out now I'm going to bring in my chief elves to check how far you've got. It looks pretty efficient over here. What's it?

Ooh, I think you've got a bit of gap there, but that one looks the closest. But let's check; my elves are going to add them up, chief elves, being a mathematician I'm useless as mental arithmetic, so I need my elves to do it for me.

Woman 138.

Marcus 138 on this side and...

Man 148.

Marcus 148. So it's the green elves that are going to Christmas Eve, big round of applause. Red elves next year. You can go back to your seats. 148, that's fantastic, if you would like to go back to your seats – that's great. 148, so very, very close. Pretty impressive. 148, but not exactly 150. Now, in fact, there is one unique way to stack these parcels in so you can get exactly 150. So it's a real winning combination. And it starts with this little box here, number 16. Number 27, here we go, that's two powers of prime there. OK, 42, the meaning of life, the universe and everything. Right we don't want 59 in there, we want 65, and that should fit very snugly inside there, and this exact fit of 150. But finding that combination is extremely difficult. There are literally thousands of combinations we could have had here, which our elves were trying to find in 30 seconds. But trying to find this one is a bit like trying to find a needle in a haystack.

It's not just Santa who was interested in this kind of problem. Actually many real-world problems in industry and telecommunications depend on finding the most efficient packing like this. After all, wasted spaces cost money, so it's not just Santa who's interested in it, in fact some codes also depend... if you want to crack a code you have to find this unique way of packing the boxes. We're going to find out tomorrow night a little bit more about the uncrackable codes that mathematics can deal with. These problems are so important that a businessman in America has offered a million dollars to anybody who can unravel these problems. The million dollars is there for the first person who can sort out whether packing these boxes, the only way is to try by trial and error, and or perhaps there's some efficient way to crack this problem. Anyway, until next time, I'm off to make my millions by applying some of mathematical secrets in the quest for the winning streak. Thank you.