Christmas Lectures 2006

THE NUM8ER MY5TERIES –
Presented by Professor Marcus du Sautoy

LECTURE 2: THE STORY OF THE ELUSIVE SHAPES

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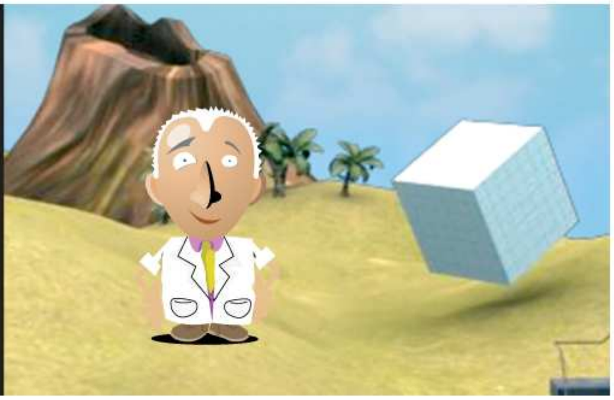

Part 1

Marcus Hello, and welcome to The Number Mysteries. If you haven't already guessed it, tonight's lecture is all about shapes, and perhaps nature's most perfect shape is the one I arrived in the lecture to, and the one you've been filling the lecture theatre with: the sphere. But why does the bubble like the sphere so much? And what other shapes does nature like? Well, in this programme, we're going to explore the rich and varied world of shapes, from the small objects like a virus, to the big shape of the universe, and, just as in last night's lecture, I'm going to tell you about a mathematical puzzle for which you can win a million dollars if you can solve it.

Now, perhaps all those bubbles we were blowing are spherical because you were blowing them through circles, and that's why they're actually spheres, perhaps if you blew it through something different, like a square or a triangle, you'd get a pyramid-shaped bubble or something. So I've got some people on the front here, who've got some funny shapes to blow their bubbles through, and we're going to see whether actually we can get some interesting shaped bubbles if we blow it through a square. So who's got a square there? Do you want to keep blowing? Let's see what you've got, yeah, you've got a square here. So if you'd like to blow your bubble and you can blow yours too and see if we'll get any… ooh, that's not a cube, that looks like a sphere again. What have you got? A triangle, let's blow yours, oh, still spheres again, not a pyramid. So, what is it? Ooh, you've got a star. We should get a beautiful thing here but, again, look, a lovely sphere. So why is it that the bubble, although you're blowing it through something like a square, doesn't come out as a cube, when it sort of starts out as a cube then it collapses to the sphere? Well that's because nature is very lazy, and this, the sphere, is actually the laziest shape. So the sphere is trying to find, a bit like when you roll a ball down a hill and it gets to the valley, that the valley is the lowest energy point for the ball. Well this is actually the lowest energy point for the bubble, and this shape is the shape with the smallest surface area. So it might start out as a cube, but it collapses to a shape which is spherical because that's a really lazy shape for the bubble. So actually, nature has known this ever since the Big Bang, the bubble is the laziest shape, and the one with the smallest surface area, but it took mathematicians till 1884 before they could prove there isn't another shape which has an even smaller surface area than this. Ok, that's great. You can stop your terrific bubbles there. Now, it's nature's lazy shape, but there's also another reason why nature loves the sphere because the sphere is incredibly strong! And so that's why we put divers inside a helmet which is spherical in shape. Well, who've we got in here, hello?

Tim It's Tim.

Marcus Tim, our assistant. Tim is here, dressed up, and he's in a spherical-shaped helmet because the pressure of the water underneath squeezes him as he goes down. The pressure is so much that you need a sphere-shaped helmet in order to be able to protect his head and not something like a cube, because that would collapse. Ok, we're gonna test this, and send Tim off to the middle of the Atlantic. You can dive down and we'll bring him back next Christmas and see how he's got on, so off you go Tim.

There's also another object which uses the sphere for strength, which is the egg. Because nature likes to make an egg spherical because the egg is really strong and it protects the embryo inside it. This is actually so strong that mathematics says that the curvature of this egg should actually, if I take 12 of them, support a member of the audience if they step on them. So I want a volunteer to try and test out my theory, to see whether it's actually going to support them. Yeah, I'll have you up here, great. Ok, now we're going to, let's get these eggs ready. So you're going to stand here, alright, with your skull and crossbones on there, what's your name sir?

Hugh Hugh.

Marcus Hugh, the mathematics says that if you step delicately on here, 12 eggs should support your weight. Now wait a moment, we're going to get two volunteers to lift you gingerly on, and I'm going to go down here and see whether there's, anything's going to break. The mathematics said it should support him, are you ready? Anyone hear any cracks? Let him go, excellent. I can't hear any cracks, ok, if you lift him off quickly, and let's see how we've done, has the mathematics proved right? Wow, that's amazing, look it's supported your weight. If we see this, there is nothing broken inside here: 12 eggs can support the weight of one boy. Now you might think I probably cheated don't you, and you think these are hard boiled. Shall we test this out and see whether they are actually hard boiled? No, no, I'm not going to do that! Ok, you've got a bowl here for me, just to show you that's actually the strength of a sphere which has actually held his weight, there you go, they were real eggs after all. Let's thank our volunteer for testing out the strength of a sphere, thank you.

Actually nature is full of spheres; she's very good at making spheres. Now you might think that a raindrop is actually kind of tear-shaped, but that's actually just because animators like to draw them as a tear to give them a sense of motion as they are going down. If you take a picture of a raindrop as it drops it's actually a perfect sphere. Now there's another thing which is a perfect sphere, it's the one you've all been coughing and spluttering during the lecture. It's actually a virus. And here's a model of a virus, much bigger than the one you've been coughing and spluttering. Maybe I shouldn't touch it else I'm going to get some dreadful disease. I'm not sure which one this is. So nature is incredibly good at making spheres, but we humans find it much more difficult.

Now, I play football on the Hackney Marshes every Sunday and we like to play with something which is as spherical as possible. In fact many sports need a really spherical ball to play with. So how do manufacturers make something which is nice and spherical? Well they like to start with something which is flat, because they can cut out bits of leather and sort of stitch them together. So we want to make something with a lot of symmetry, so let's start off with some shapes, try and piece together some shapes with a lot of symmetry. It was an Ancient Greek mathematician called Plato who first started to investigate what sort of shapes you can make for footballs and we've got some of them here. Ok, let's start with a symmetrical shape like an equilateral triangle, so I can take an equilateral triangle, very symmetrical shape. Actually, Plato realised you could put four of these together to make something called a tetrahedron, this is quite a symmetrical shape, but not very spherical. I'm not sure this ones going to make a terribly good football, do you? Let's test it out. I'm actually a central defender for my football team so they don't let me take free kicks. I'm going to try and score a goal right at the back there, but I warn you the people at the front this may, you know, it's also not a very good sphere, so I'm going to blame it on the ball, you know, a good workman blames his tools. So let's see where this one's going to go, see if I can score a goal with this Plato football. Well, not a goal, but I took one of their defenders out, so I think your wall worked very well. Ok, well, I can blame that on the fact that it wasn't a terribly good spherical shape, and we've got some better ones. Actually, Plato realised that if you took equilateral triangles, actually, you could take eight of them and piece them together to make what we call an octahedron, so that's getting a little bit more spherical shaped. But here's another one, very symmetrical shape, the cube. Of course we can stick six of these together and that makes something looking a little bit like a dice and so the dice is very symmetrical, and that's why, you know, it's a very fair thing to play a game, but it's still not very spherical. Well, this one's quite good actually, because this one's made out of pentagons, and a pentagon is starting to look a little bit more like a circle. That's a very sort of circular shape maybe it makes a good sphere? That's not bad, actually Plato realised you could put twelve of these together to make, actually they used to call it the 'sphere of 12 pentagons', they also called it a dodecahedron. So let's try that one out; that might be a bit better. Who can I take out now? So this is a bit more spherical, I think I might be able to score a goal this time. Are you ready? Hey, that's not bad, that's not bad, top left-hand corner. Now Plato discovered there are only five different shapes you can make and the last one is this one here, we're starting again with the equilateral triangle but you can put 20 of those together, we've seen one with 4, one with 8, but this one is actually with 20 triangles together and this is called an icosahedron. But Plato proved that there are five, and only five, shapes you can make by putting these together. I can't take any other combination of triangles and make a different shape.

So this is getting pretty spherical, but there is something I could do to make it a little bit more like a ball. I could cut off some of the corners here. Let's try and cut off the corners and see if we can make a really good football, let's put that one back here, so I've got a shape over here, it's our icosahedron, started off as that, and you see all the triangles on here. Now what I'm going to do is cut off all the corners and try and smooth it out and make it a bit more like a sphere. Ok, let's take these off. Now, when I take off the corners, you see what happens, the triangle has actually become a hexagon, as I take off these, the corners, interestingly, become pentagons. So here's another one, take that one off. If I take all of them off, there are 12 corners there, take them all off, actually I get something which looks very familiar, that looks like the football I kick around on the Hackney Marshes every Sunday. Now this one's quite interesting, it's made up of two different shapes, we've got the hexagon here, but we've also got some pentagons. Now I've actually got an unwrapped football for you to see. This is what it starts off as, something very flat, all these flat pieces and they sew them together. So remember you had 20 triangles, I cut them up, so I've got 20 hexagons here, you can see the bladder underneath there, and we've got 12 pentagons. And if you sew those up you get the classic football that we kick around on a Sunday.

Now this shape is made out of a combination of symmetrical objects, so I wonder what other footballs we could make. Well it wasn't Plato, but another Greek mathematician called Archimedes who explored the different sort of shapes you can make if you take symmetrical shapes, but lots of different ones, so here they are. Archimedes proved that there are only 13 different footballs, in addition to Plato's, which you can make. There are quite interesting shapes here, for example, let's see something interesting. Yeah, this one I quite like because if you see here, this one's got squares, it's got hexagons and it's also got octagons inside here. So that's combining three different shapes, now all the shapes actually have the same edge length, but you can still piece them together, and there are only 13 different possibles, no more other than these 13. So, and I think we've got the football here, there's the football, now maybe next World Cup we're going to see one of these different footballs, maybe we're going to get this one. This is a snub-dodecahedron with 92 different faces. So maybe World Cup ball of 2010 is going to be something like this.

Ok, I've told you a little bit about the symmetrical shapes of nature, but in the second part I'm going to tell you a little bit about how mathematics can build nature's very unsymmetrical shapes, things like the coastline of Britain. The coastline of Britain is a very unsymmetrical shape, so how can mathematics tell us about that? Well, here's a little challenge for you during the break, I'd like you to estimate how long do you think the coastline of Britain actually is? So do you think it is a) 18,000 kilometres, or do you think it is b) 36,000 kilometres or do you think it could be c) infinite in length? Come back after the break to find out.

Part 2

Marcus Now, before the break, I asked you to estimate how long you thought the coastline of Britain was. Now this turned out to be an actually more complicated question than you might first think, and to explain, I'm going to need a volunteer. Who's going to try and help me measure some coastline of Britain? Ok, yeah, so if I could have you, now why don't you come up, sir? Right, we've got a bit of coastline of Britain here, and we're going to get this young man to help me measure it. We want to stand here, great. And, so what's your name, sir?

Rory Rory.

Marcus Rory, you're going to measure some coastline of Scotland. Actually, there's a bit of Scotland, so you're going to hold there, if you go that side, great, and what we're going to first do, is use the piece of string to measure the length of the coastline. If you could take one end of the string here, this is going to give us some sort of estimate of how long the coastline is. What's that, about a couple of metres or something like that? Ok, but that didn't really get into all the inlets in this bit of Scotland. So what we're going to do, there's another piece of string inside here, which is much thinner. If you want to take that end here, we're going to pull this out, this gives a much better and more accurate measurement because I can get inside all of these little coves and things like that, great. So if you want to hold both ends of your string; and the thin string and pull it out there, we're going to compare this, hold onto both of them. Now the first string gives out about here, but with the much more accurate measurement I can get a string which is about twice as long as the first one. So it looks like it depends how carefully you measure the coastline of Britain, how long it's going to be. Let's give our volunteer a big round of applause for helping to measure that, great. Now, actually, the Ordnance Survey map says that the coastline of Britain is actually 18,000 kilometres long, but if they'd measured it a little bit more accurately, they might have got double the length at 36,000 kilometres. But if I keep on measuring in more finer and finer detail, had an even thinner string than this one, perhaps I'm going to get this coastline getting bigger and bigger and bigger. It could actually be infinite in length. To see why that might be possible I'm going to need somebody who's going to build a little bit of mathematical coastline for me. Yes if you'd like to come up, sir. So we're going to put you behind here, and we're going to build a little bit of mathematical coastline. Ok so what's your name?

Tommo Tommo.

Marcus Tommo du Sautoy, in fact this is my son, I'm afraid to say I pulled him out. Ok Tommo, you're going to pull and make a little bit of mathematical coastline for us, and what you're going to do is going to, so this looks a bit straight this coastline here, it doesn't look very nice. So we're going to make a little inlet here, we're going to pull this bit of the string in the middle here and I want you to hook it around this top bit here, great. Ok we made a nice little inlet there, great, but now we're going to keep on repeating this, every time you see a straight line what I want you to do is to add on another little triangle. We're going to start down here put a little inlet into this coastline, great, that's one, now here's another straight line, if you could put a… so we're going to take a little, take the middle. What we're doing is repeating the mathematical rule which is add a triangle every time you see a straight line, so pull that one, great; pull that one, excellent. So now we've added some more smaller triangles, but we could keep on doing this, every small bit of length of string I could add an extra little triangle and if we keep on doing that, here's a shape we got prepared a bit earlier. Let's show them this shape, pass it on. Ok, so if we keep on repeating this we're going to get an ever more complicated sort of bit of mathematical coastline. But could this actually be infinite in length? Well let's give Tommo a round of applause for creating a bit of mathematical coastline for us.

So, this is getting ever more complicated. In fact if I zoom in and add more and more triangles, we've got a little animation to show you how infinitely complicated this shape is actually going to get. You see us zooming in; you can see all the finer, smaller and smaller triangles. I'm just adding on more and more triangles off to infinity. Now this is actually a very special shape called a fractal, and fractals have the special property that they can be infinite in length. So let me try and explain to you why this shape, with all its infinite amount of triangles could be infinite in length. It's only fitting on a finite piece of paper, so how could it be infinite? Well, what I've got is a little small piece of the coastline, this mathematical coastline, in fact this is this coastline reduced by a third. So if I take this coastline, shrink it by a third, so here's the long line, shrink it by a third, I'll get this small piece here. So the big coastline is actually three times this length here, but hold on, actually if I take this coastline I can fit it in four times into this length. I can put it once here, a second time here, a third time here and a fourth time there. It seems like this coastline is three times this one, and also four times this one. So have I proved that three equals four? That seems pretty bad, perhaps the whole of mathematics is going to collapse now, but now the answer to this is that this is actually infinite in length and three times infinity is four times infinity. So actually fractals can give you an infinite length. So coming back to that question about how long the coastline of Britain is, well actually it can be all three answers. That's the kind of maths exam I like, where any answer you put down, it could be right, it just depends on how you measure the thing, how accurately you measure it.

You can see it produces… now this piece of coastline it's really not terribly natural, it looks incredibly symmetrical doesn't it? So how can we actually produce something that looks a little bit more natural? Well there is something we could have done on this. Tommo, instead of pulling out of the small triangles upwards, he could have randomly chosen to pull some up and put some down. So, for example, this triangle, we could have decided to randomly put it down, and then we start to see something which is a lot more natural. So what I've got up here for you is a shape where we've got some triangles, and I'm going to randomly pull them in and out, and if we continue doing this to make a fractal random shape, by the time you finish this thing, you get something which looks remarkably like a sort of medieval map of Britain. So that's the reason that a medieval map and the real map could actually be infinite in length. Now, so these infinite fractals have this wonderful property that if you zoom in on them you get this infinite complexity, and that's what so special about a fractal. Actually nature is full of these fractals and I'm going to show you about this infinite complexity by unleashing one of nature's most deadly forces.

I'd like you to give a big Royal Institution welcome to the Prince of Darkness himself, Russell Thomas from the National Physical Laboratory. Come on down here, so welcome, welcome Russell. Now Russell is actually going to create some real lightning for us inside the lecture theatre. We had this sort of theatrical stuff, but I want some real lightning. So, when Russell is ready getting all his equipment here, we're going to get some real lightning. How on earth are you going to do it Russell? What are you going to do for us?

Russell Well, what we've got at NPL is a linear accelerator, one of the beams that we can produce is an electron beam, and we can use that to charge up these acrylic blocks, and I've stored this in dry ice.

Marcus So this thing has been charged up with loads of electrons?

Russell Yes, we did that yesterday, cooled it down and hopefully now we're going to try and get that charge to come out.

Marcus Right, so this is a bit like some sort of spring then.

Russell Yes.

Marcus And we're going to kind of release it, and then we're going to get some lightning. Now is it going to be dangerous? Should I hide behind there, being a bit of a coward?

Russell No, you're ok.

Marcus Oh right, alright, let's see if we can create some lightning then inside the lecture theatre.

Russell It doesn't always go the first time.

Marcus Ok. Give it a second go.

Russell Doesn't always go the second time.

Marcus Yeah, third time lucky, prime number you see. Well, lightning you also can't predict. Oh wow, now that's fantastic and it is still going actually, look at that, all of these are electrons shooting round, that's really beautiful. Now it's quite hard to see this one, you can probably see the crackling. Can we make another one? I think that was quite exciting. Have you got another one?

Russell I have, I've not ever tried it with this size before.

Marcus Oh right, oh my god, you're going to make a really big piece of lightning. This looks really terrifying. I think I might hide behind here.

Russell Let's see if we can get this one. Third time lucky.

Marcus Wow, and all those sparkles again, that's amazing. So actually we've managed to capture in this acrylic some lightning, absolutely extraordinary. Now, Russell, actually we've got one that you prepared earlier which we can light and we can really see the beautiful fractal shape here. Let's put this one up here, so that is actually what we've made, we've actually managed to capture, this is lightning inside this acrylic and you can see the sort of infinite complexity in this thing. So if we zoom in on this shape, actually it looks like it is very hard to judge the distance from it. Are you zooming in very close? Or are you far away? See we've got these branches here coming out and then we've got much thinner branches, thinner, thinner, thinner. So this really looks sort of like a fractal like we created on the landscape. Let's give Russell a big round of applause for creating some fractal lightning, great. Thank you very much Russell.

So lightning is fractal, but actually nature is full of lots of fractal shapes and we've got some over here, for example if I take this fern. Now fern, if you look at it far away and zoom in very close, actually it's quite hard to judge the sort of distance you're at, it is sort of infinite complexity here, the little fern looks like the big fern. In fact, inside our body we also have some fractals. The human lung is also fractal in shape. So if you look, this is a little bit like the lightning that we saw. We've got these large branches here, but with smaller and smaller branches coming off, and it has its infinite shape of the fractal. Now actually these natural organisms are using the infinite length that a fractal has to maximise the amount of oxygen these things can take in. But it isn't just nature that likes making fractals. In fact, fractal geometry is also the key to understanding one of the most expensive pieces of art ever sold.

Jackson Pollock is an artist who used to create his art by flicking paint around. Last month one of his paintings sold for a staggering £73 million. Now I've got three volunteers who are ready and primed to make us a Jackson Pollock. So if you'd like to come up, give them a big round of applause, they're going to make some. Great, come on down here. If you'd like to go behind the red paint, hey, great. So we're going to get them to flick some paint around like Jackson Pollock, so over here you're going to have the yellow paint. So what's your name, sir?

Kenneth Kenneth.

Marcus Kenneth is going to be on the yellow paint, so he's going to flick it around to try and make a Jackson Pollock. And over here on the red paint, who have we got?

Genevieve Genevieve.

Marcus Genevieve is going to be on the red paint. And on the blue paint to create our Pollock is?

Isla Isla.

Marcus I love?

Isla Isla.

Marcus Isla is going to be doing the blue paint and we're going to set them off to make a Jackson Pollock and hopefully you're going to become incredibly rich, £73 million please is what I want out of you. Ok, we're going to set them off. Now last month actually a lot of, off you go, you can dip your paint in. I'm going to keep well clear, try and keep it on the, that's excellent, oh my god, keep clear of this! Right, they're going off there.

Now a lot of people said last month, when this Pollock sold for £73 million, that, actually, anybody could do this, you know, what's the big deal about Jackson Pollock flicking a load of paint over? Well, unfortunately, not anybody can do this, because when mathematicians started to study these Pollocks, they realised what he was doing when he was flicking his paint around, was actually something quite unique. Here's a Jackson Pollock up here and it has this fractal character, that if you take a Jackson Pollock from far away and zoom in, it's quite hard to judge the scale of the Pollock. So it seems like Pollock was actually doing something very special and creating some sort of fractal shape when he was flicking. So let's see whether, I don't think this has got the infinite complexity. Can I have a go? This looks really fun actually; I'm going to do a bit of blue paint. So, he was sort of doing some sort of flicking motion, where he got some sort of fractal coming in here, so that if you zoom in on this, it will have this sort of infinite complexity. In fact, there were 32 Jackson Pollocks that were revealed last year to be fakes, by using the mathematics of fractal geometry. Ok, well let's give our volunteers a big round of applause for creating something rather beautiful. So, to the untrained eye this might look like a Pollock, but mathematics can tell us that it isn't. Fractal patterns like this appear all over nature. They look very complicated, but actually the mathematics is actually quite simple, that you repeat to create something this complicated.

Now we've seen so far some of nature's lovely shapes, like the sphere, the beautiful spherical sphere, also these fractal shapes which appear in nature, but in the next part, what we're going to go for is some really big shapes. In fact, I want to understand the possible shape of the universe; and, as a little warm up to exploring the shape of the universe, here's a little question for you to think about during the break. Some of you may have played the classic game Asteroids, where your spaceship shoots off one side of the screen and appears again at the other, but what I want you to think about is what is the actual shape that our two dimensional spaceman is travelling around on. Come back after the break to find out.

Part 3

Marcus I left you, during the break, trying to work out what the shape of this spaceman's universe is. Now he's actually travelling around. It isn't infinite because it's this little rectangular box, but it seems to be joined up in some way, because when he goes up here, he comes in at the bottom. So I want a volunteer to try and find out what the shape of this universe actually is. Yes, would you like to come up? We're going to kind of try and find out what shape this really is by joining it up somehow. So I've got a little universe down here, if you'd like to come, you stand this side, and I'll come this side. So what's your name?

Anna Anna.

Marcus Anna is going to try and see what the shape of this real universe is like. So let's hold it up, here's our screen, this is what the game looks like, so when the spaceship goes off at the top here, like at number 2, it rejoins again at the bottom here. It seems like these two bits of the universe are actually joined up. Similarly, if it goes off here at 1, it comes on again at the bottom at 1 here. So I think these two bits are joined up and we should actually connect them. Let's hold them down here, let's join the top of the screen and the bottom of the screen together like this, smooth it down like this. Actually the universe has started to look rather cylindrical. But there's another direction that the spaceship can go off in. It can go off to the right of the screen, it might exit at A and then it comes on your side, at A, like this. So it looks like this bit is joined to here, and again, if you see, there's a B here, and that's joined to here, so it looks like we're going to need to sort of bend this round to join the two pieces up. Now this one's made of card, so it's quite hard to bend up, but we've got one down here, so if you'd like to help me lift up one that we've bent round, and, actually, here is the shape of the universe, but our spaceman is travelling around it. It's actually something called a bagel, that's right, a bagel, or one of those Homer Simpson doughnuts, or what we call in the trade a torus. Ok, let's give our volunteer a big round of applause for making our space, and, look, here's a bagel for you as a reward!

So when you're playing one of these computer games, this is actually the shape that you're playing on. In fact, I've got an animation, to show you how this space is wrapped up. Here the spaceship's whizzing around, and you see the top and the bottom of the screen are joined up. So we take those round, make a cylinder, then if we join the cylinder round, the two ends are joined, because when you go out of one screen, you come out the other side to make the shape of the torus or the bagel. So that's good, we can see that shape. Because we're three dimensional, we can sort of see how it's folded up and we can see the hole in the middle, for example. But how about a two-dimensional spaceman who's stuck in this space? He can't go outside to see what the nature of his space is, so is there any way that a two dimensional spaceman, stuck inside this world, can work out what sort of shape of universe he's in? Well there are quite a lot of possibilities. He could actually have been in an infinite kind of space, where he just shot off into infinity and didn't come back again, or he could be on this bagel, or he could be on a globe, going round. I heard somebody, during the break, saying you thought it might be a ball-shaped universe that he was in. So is there any way that this spaceman can tell which of these two shapes he might be on, by travelling around and doing some exploring? Well I'm going to explain a couple of ways, and I'm going to need some volunteers to try and help me find one way. So let's go over here. If you'd like to come up, sir? Alright, you're going to be in charge of this sphere-shaped world over here. So what's your name, sir?

Mahanesh Mahanesh

Marcus Mahanesh is in charge of this ball. Now, I'm going to take somebody else, from over here, yes, if you'd like to come up. Why don't you come up? Great, she's going to get the bagel, ok, so come on down, right, gosh, mathematics here is quite an exciting subject. So you've got the bagel, so what was your name?

Lisa Lisa.

Marcus Lisa, ok. So, you've both got shapes and you're going to… you can see they're furry shapes, so what's the fur got to do with this? Well, what I want you to do is to take a comb each – there's a comb for you and a comb for you – and what I want you to do is to try and comb the fur on these shapes so it all lies flat in one direction. Off you go, we're going to let those go, you can kneel on the floor if you need to, or whatever. We're going to leave them trying to smooth down the fur so it all runs in one direction and I'm going to show you a second way that you can tell which shape you're on. For example, if our spaceman was stuck on this globe, let me show you something rather special about this shape. Now our spaceman might be up here, at the North Pole, and he could do a journey, he could go straight in this direction, so here we go, he's going straight down, making this journey, all the way round to the South Pole, back again. And the thing about these shapes are they're closed up so he comes back to the beginning again. But what he could have done is set off in a different direction. What if he set off at 90 degrees and went round this way? Ok, let's send him off, round, look he meets his first path down at the South Pole, and he keeps on going straight, this is the straight line in his world, and he comes back to the beginning again. So, on a sphere, a journey by a spaceman will actually cross the first journey he made twice, once at the South Pole and once at the North Pole. But there's something rather different about a bagel. Let me show you some journeys on a bagel. Let's put that one here, on a bagel you can do some different journeys which don't meet, because our spaceman could have started up here. Let's set him off in a straight line, so he's going straight round here, comes all the way round, finds himself coming back to the beginning again. He could have set off at 90 degrees to himself, on the sphere he met the same path twice, but this time he goes inside the bagel, whoop, let's turn it round the other side here, comes back out here and comes back to the beginning again, where's the beginning, here we go, without having crossed the first path at all. And so, actually, by making journeys inside his space, he can tell that he's on a bagel, because there are two different directions he can go in, which don't actually meet. But on the sphere they will always meet. So that's a way you can tell with the spaceman in 2D, without going outside his space, that there's a hole in the middle.

Now my two volunteers here have, I hope, found another way. How are you getting on smoothing your bagel? Ok, it looks pretty good, yeah, we're smoothing it, you seem to be doing it all in this direction, is that right? Actually you see, if I keep on going, I can smooth this all the way round, sweeping round, to make it perfectly smooth and there's no bit of fur actually standing up, so you've done pretty well there, you know. It’s a sort of celebrity hairdresser or something. There we go, but on the ball, on the sphere here. Hmmm, you’re doing pretty well here, but if you see that, there’s this little crown here, do you see the crown? It’s very hard to smooth it down isn’t it? Sometimes you get it smooth here and this is actually a property of the ball, you see there’s always going to be a place where somehow the hair is sticking up. Actually we find that on the head, let’s have a look at your head. I see here, yes actually you can see the crown here, sorry about this, but let me take my big comb and try and comb his hair. But there’s always going to be a space where all the things sort of aren’t lying down; this little tuft comes up. So this is called the crown and actually the bagel, there’s nowhere where you get this crown, but on the ball there’ll always be one place that you can’t smooth down. That’s another way that our spaceman could tell which shape he’s on. Ok, let’s give our volunteers a great round of applause, thank you.

So we’ve started to explore the two-dimensional world, we’ve kind of warmed up with that, but what I’d really like to find out, is to explore our three-dimensional universe. Now the 2D spaceman can move in two directions, he can go right, left or up and down, but I’ve got another direction I can move in. But, like the spaceman I’m stuck in my three-dimensional world and I can’t actually go outside to actually see what shape I’m in. I can’t see whether our universe has a hole in it. But perhaps we can use some tricks that we’ve learnt here about how the 2D spaceman can work out what his shape is without going outside to try and explore what our 3D universe is. Now what sort of possible shapes could it be? I think most people have this sense that our universe is probably infinite and it goes off, you can go off into space and you’ll just keep on going off into infinity. In fact more scientists now think that the universe is finite, but doesn’t have any edges, so that if I shoot off in this universe, somehow I come back to the beginning again. So it’s a little bit like that game of Asteroids where you keep on going round and round and coming back to where you were. Ok, if I’m really going to explore the shape of my universe, I think I’m going to need a rocket to do it in, so I wonder, Andy, could you bring on Beagle 13? Ok, fantastic, right, nice careful landing, great. Now I think Wallace and Gromit would be proud of this spaceship. So, I want to explore the shape of my universe. Now, perhaps it’s a bagel like the one we saw our two-dimensional astronaut was flying around in, but this isn’t a three-dimensional bagel, but a four dimensional hyper-bagel. Now how could I tell whether I was living in a four-dimensional hyper-bagel? Well, you remember when the spaceman went off this side, he came on again here. So perhaps I should try out and see what happens if I go off this side of the lecture theatre. There’s only one way to find out, which is take my rocket and see where I end up. We've come on again over here, so looks like my universe is somehow joined up. Ok, let's try another direction, why don't I shoot off here into the audience? I wonder where I come on now. Let's give it a go. I come on again at the back of the audience. Now this is beginning to look like that game of Asteroids: go off this side come back there, go off that side come back in here. Ok, but I'm a three-dimensional astronaut and I've got another direction I can go in to. Let's see what happens when I go up. Ok, I'm going to need to get in this spacecraft so Andy, can you give me a hand here, great, right, let's turn the porthole round. I'm all ready, let's see where I end up if I shoot off into the ceiling. Three, two, one, blast off. Well it looks like I've come through the floor. So the shape of the universe we've created in our theatre is actually a four-dimensional bagel. But what else could it be?

That's this lecture's million dollar problem. Could it actually be a four-dimensional football, or one of these hyper-bagels, or something even more exotic? Well, till next time I'm off in search of the elusive shape of the universe. Goodbye.

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