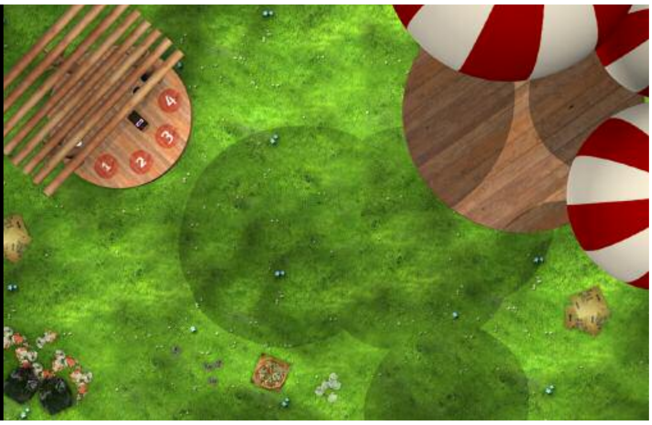
Information salvation



Computers are essentially electronic machines for handling information: storing, processing and transmitting it.

Computers do two very important things with information: *compress* it and reliably *communicate* it. For example, you can fit so much music onto your MP3 player because the songs have been *compressed* to use up less memory. The computer identifies patterns in the music track and saves it in a more efficient way, without losing information that is important to the way the music sounds. Every time you chat to a friend on your mobile phone you use digital *communication*.
A computer chip inside your mobile converts the sound of your voice into a stream of digital information that is transmitted to your friend's phone where it is turned back into sound.

The engineers that design gadgets like MP3 players and mobiles and the software that runs on them need to understand *information theory*. This kind of mathematics lets us calculate how much information something contains and what is the best way to compress or transmit it. You may not have realised but information theory is behind a lot of modern life: TV, the internet, CD and DVD players, as well as communicating with deep space probes exploring other planets. Scientists also use information theory to work out how our brain transmits information between all the nerve cells!

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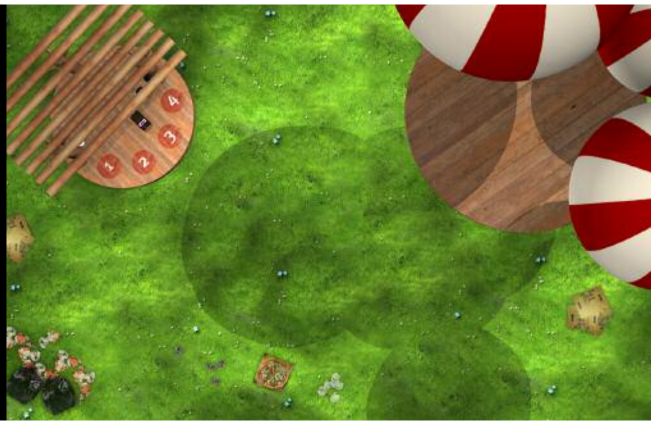

WHAT IS INFORMATION THEORY?

Information theory can be understood by thinking about how unlikely the outcome of an event is. For example, telling someone the result of flipping a coin conveys less information than the result of rolling a dice. A coin flip has two possible outcomes (heads or tails) whereas throwing a dice can result in any one of six different numbers. This means that information theory has lots in common with probability and statistics.

You can use information theory to work out the best way of finding out information. For example, say I'm thinking of a number between 1 and 10 and you can ask me only 'yes' or 'no' questions to find it. How do you work out my chosen number as quickly as possible? In other words, which questions should you ask to get as much *information* from my 'yes' or 'no' answers as possible? Asking 'Is it one?', 'Is it two?', 'is it three?' is not a good strategy because any one of the numbers between 1 and 10 is not likely to be the correct guess. So you won't gain much information from my answer. However, if your first question is 'Is the number five or less?' whichever answer I give you can really narrow down your search. My answer has conveyed the maximum information because the probability of me answering 'yes' or 'no' is 50-50. By asking this question, my answer removes the greatest amount of uncertainty and you get the greatest amount of information (uncertainty and information are the opposites of each other).

The important thing to remember about information theory is to think about how to ask questions so that the answers are as likely as each other – so you get maximum information from the answers, no matter what they actually are.

Can you use this understanding of information theory to solve the odd-apple-out challenge?

THE ODD-APPLE-OUT CHALLENGE!

You have 12 apples that look identical, but actually one is either heavier or lighter than the others. You have a set of balance scales to weigh the apples. The challenge is to work out which is the odd-apple-out and whether it is heavier or lighter than the others, with as few weighings (uses of the scales) as possible.

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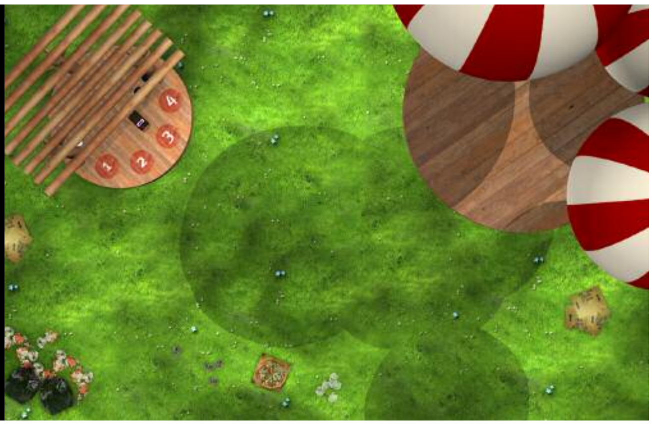
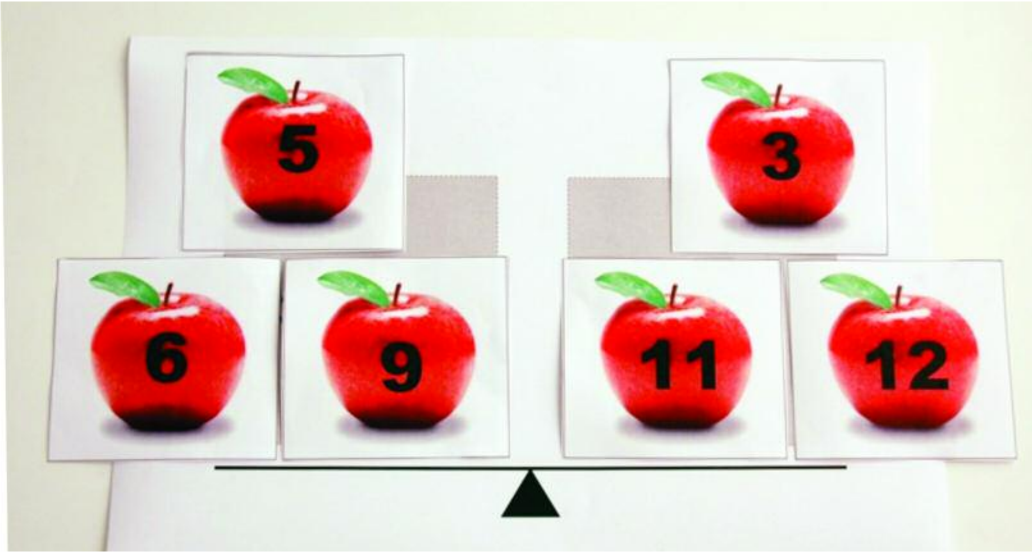


Try this challenge with your family at home or ask your teacher if you can do it at school.

1. Print out the balance scales and 12 numbered apples from this PDF for each group. Place the scales flat on a table and cut around the 12 numbered apples. One person in the group acts as apple master (you can take turns); everyone else is a challenger and has to try to find the odd-apple-out in as few weighings as possible.
2. The apple master randomly chooses an apple and decides whether it is heavier or lighter than all the others. They then record the apple number and whether it is heavier or lighter on a piece of paper and keep it to show the challengers later.
3. The challengers try to work out which is the odd-apple-out. They can weigh apples by putting whichever ones they choose onto the scales. They can put on as many or as few apples as they like, but must always put the same number on each side.
4. The apple master looks at where the odd apple is (and whether it is lighter or heavier than all the others) and tells everyone either ‘left side heavier’ or ‘right side heavier’ or ‘scales balanced’ as appropriate.

For example, the apple master chose apple number 3 to be heavier. In the weighing shown in this photo she would say ‘right side heavier’ because the apples on the left-hand side are numbers 5, 6 and 9; and the ones on the right are numbers 3 (the heavier apple), 11 and 12.



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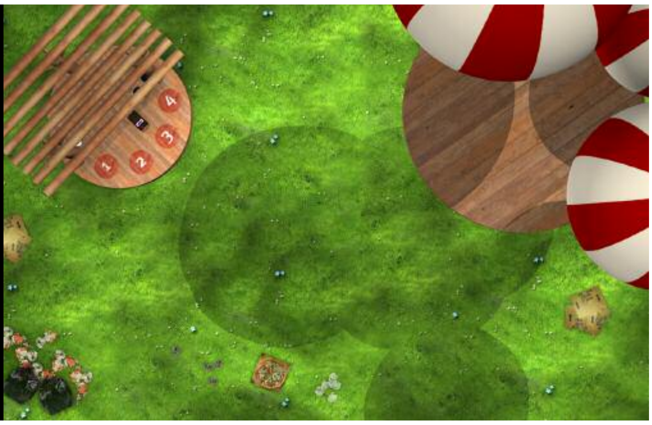
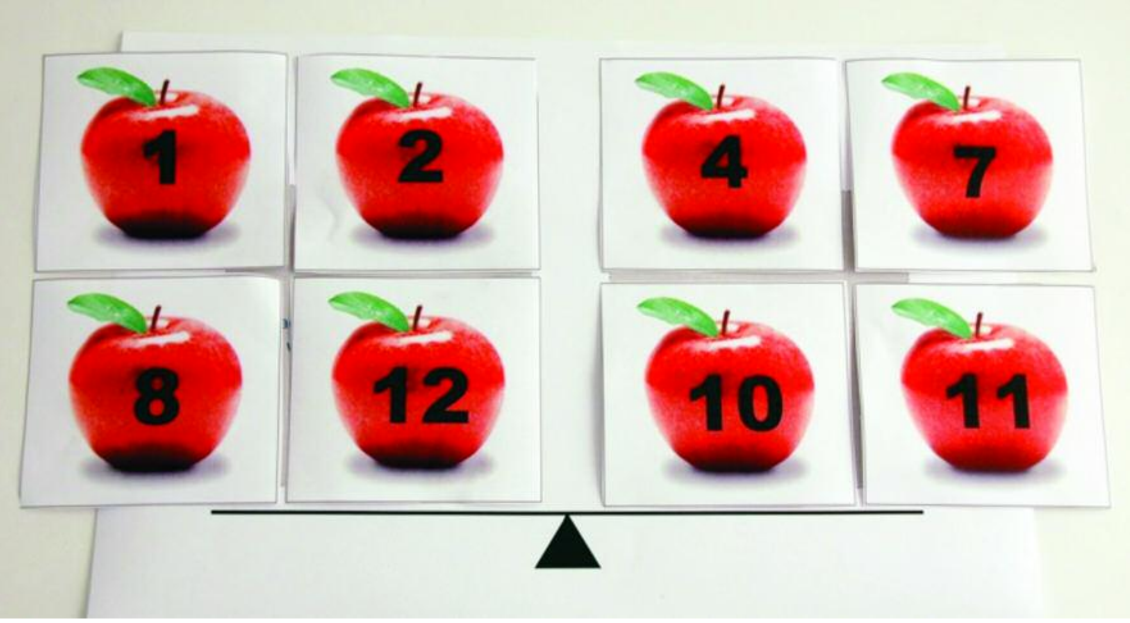


But in the next weighing there are four apples on each side: apple numbers 1, 2, 8 and 12 on the left and 4, 7, 10 and 11 on the right. Because the odd-apple-out is not being weighed, the apple master would say 'scales balanced'.



How can the challengers use as few weighings as possible to work out which is the odd-apple-out *and* whether is it lighter or heavier than all the others?

They need to use information theory to work out how to use the scales and apples to reveal as much information as possible with each weighing. They should think about the following questions.

- How many possible outcomes are there for each use of the scales? How can the challengers arrange the apples on the scales to make each outcome equally likely and maximise the amount of information they get each time they use the scales?
- What is the best first weighing on the scales?
- Once the challengers have worked out which apples must be normal, how can they use them cleverly?

The answer is shown below, but don't look at it before you've tried the apple challenge – otherwise you'll miss out on the fun!

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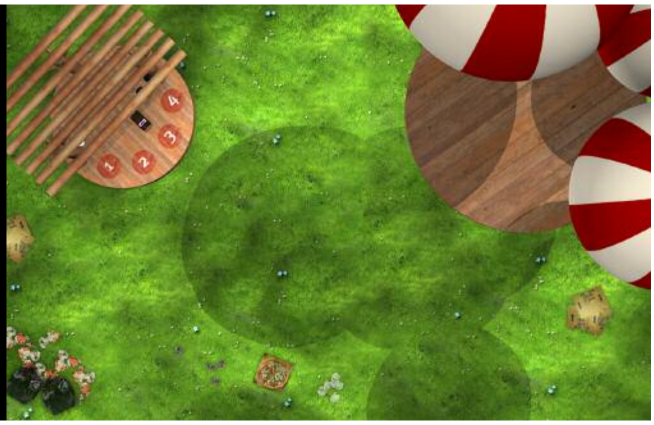

Place the apples you want to weigh on this side here

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Christmas Lectures 2008

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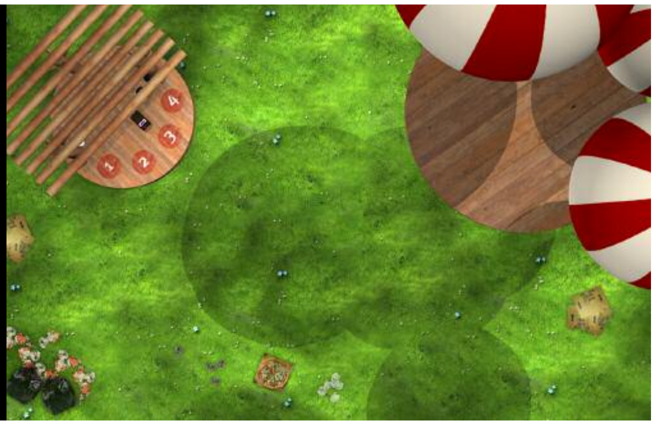
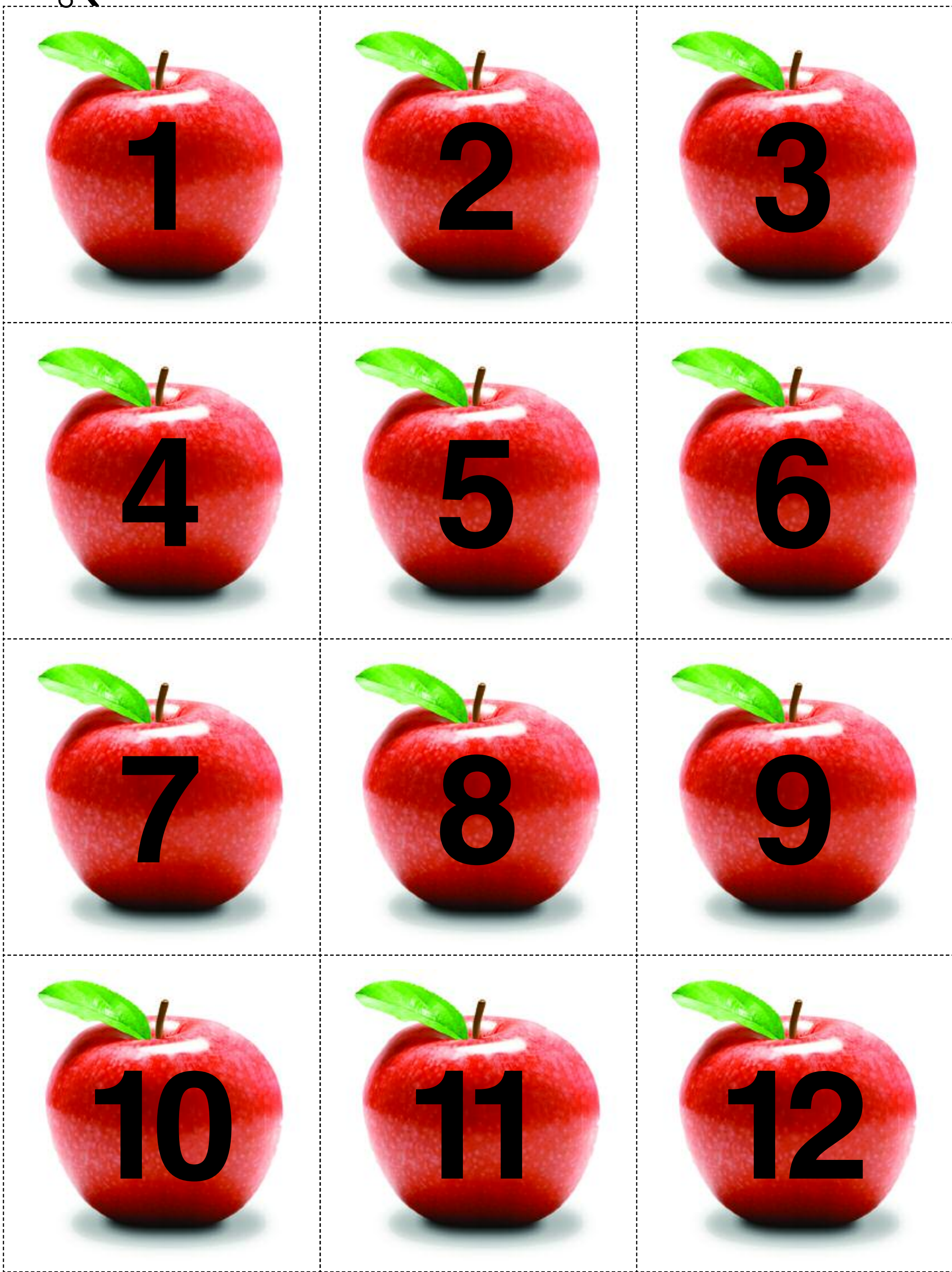
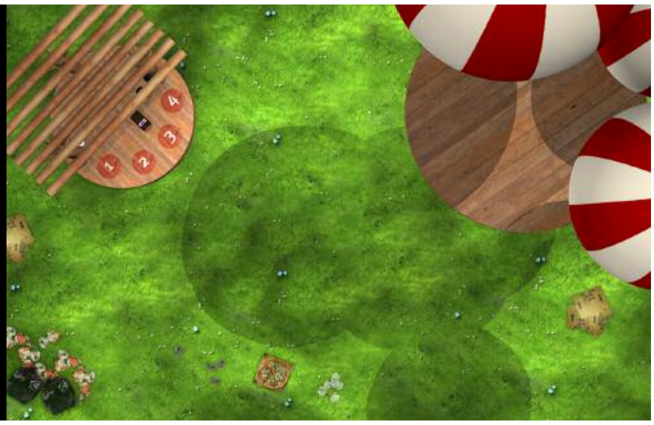






ODD-APPLE-OUT – THE ANSWER!

Whilst trying the challenge, you will have realised that there are three possible outcomes from each weighing: left side swings down, right side swings down or scales perfectly balanced. Thinking about information theory, you know that if you choose the apples carefully you can find out maximum information with each weighing – so that you expect the scales to show left, right or balanced, with equal probability.

For example, on your first weighing, you might think this seems sensible:

1_2_3_4_5_6_Δ_7_8_9_10_11_12

But there are actually only two possible outcomes. You know one of the apples on the scale is either heavier or lighter, so the scales will definitely swing down on the left or on the right – it will not balance. This means that you have only two possible outcomes and you will not get as much new information out of the weighing as you could have. The best approach is to divide the apples into three groups and try this:

1_2_3_4_Δ_5_6_7_8

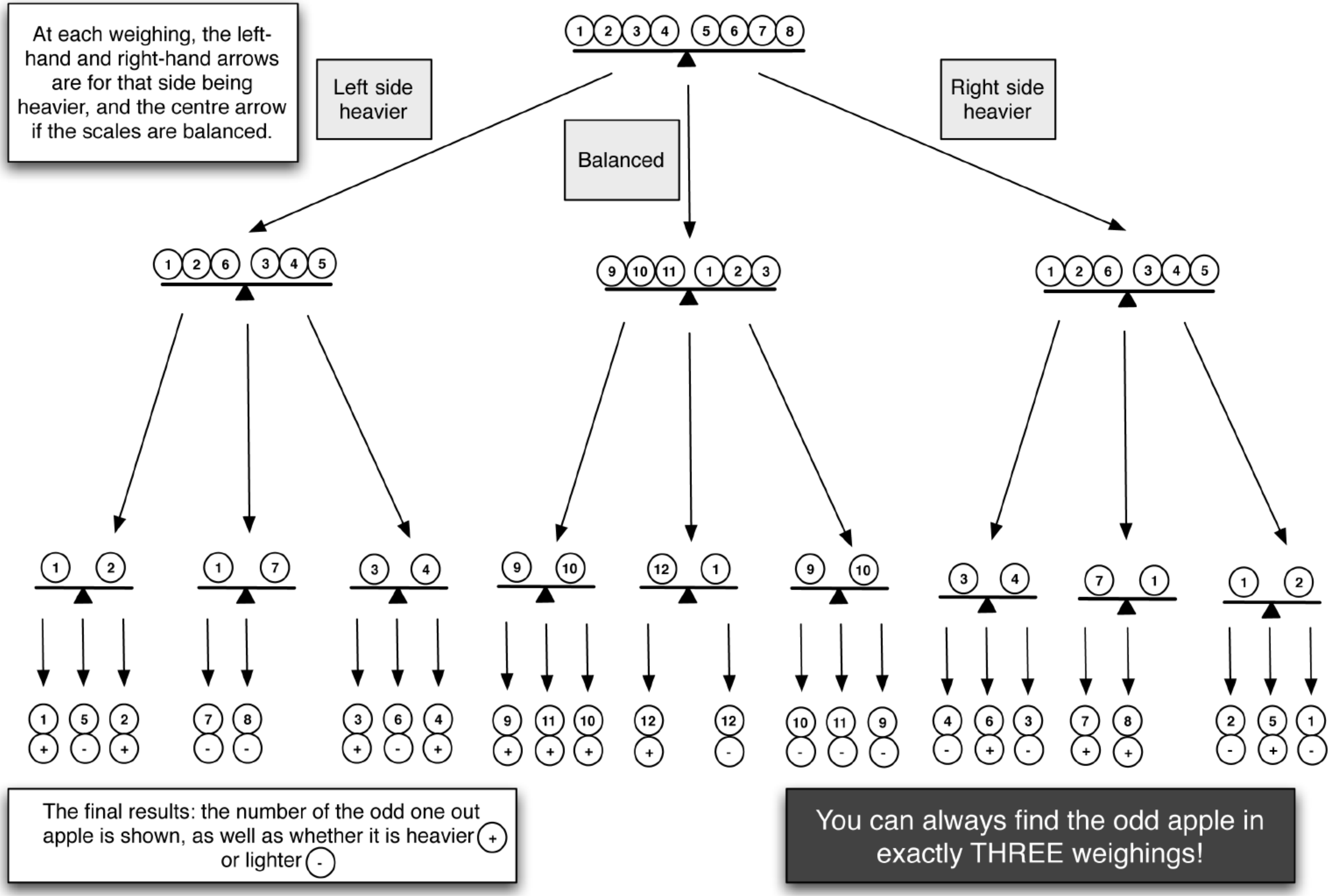
The scales could swing left, swing right or remain balanced. As each of these outcomes is equally likely (with a probability of 1/3), whatever the answer is we've found out the maximum information possible from one weighing.

The diagram on the last page is drawn from David MacKay's book *Information Theory, Inference, and Learning Algorithms*. It shows one complete solution to the odd-apple-out challenge (other solutions are possible too). It's complicated so don't worry if you didn't work out the whole solution, but the result is surprising:

You can always find the odd apple in only THREE weighings!

And you can use information theory to prove that three weighings is enough for 12 apples (with one being heavier or lighter). Here's how.

There are 24 different answers to this puzzle – 12 apples and each one possibly being either heavier or lighter. If you plan the weighings carefully, every time you use the scales you get information on three possible outcomes (left side heavier, right side heavier or scales balanced). So, using the scales three times gives you enough information to tell between 3 x 3 x 3 = 27 different situations; 27 is more than the 24 possible apple answers so information theory shows us that three weighings will be enough to find the odd-apple-out, provided you make the best decisions about which ones to weigh.


At each weighing, the left-hand and right-hand arrows are for that side being heavier, and the centre arrow if the scales are balanced.
Left side heavier
Balanced
Right side heavier
The final results: the number of the odd one out apple is shown, as well as whether it is heavier + or lighter -
You can always find the odd apple in exactly THREE weighings!