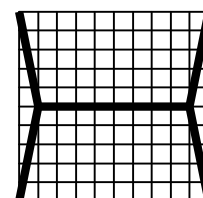
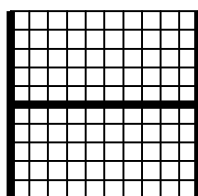
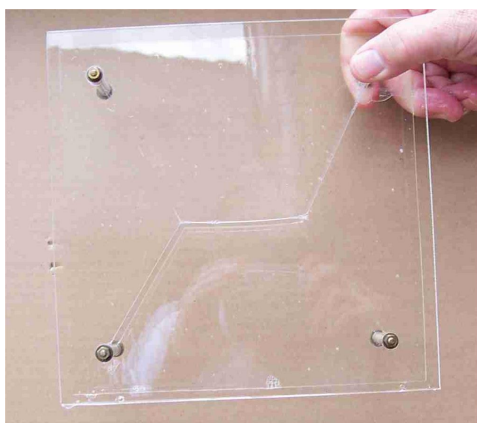




What can soap films tell us about motorways?

Teacher notes



Teachers should read through the following activity ideas and make their own risk assessment for them before proceeding with them in the classroom.

This activity draws together a range of areas of mathematics and its applications and considers how and why soap bubbles can provide the answers to some seemingly unrelated questions.

Pupils will get the chance to design and test 'motorway networks' connecting 'cities'. Through trying out different configurations of roads, pupils will make and refine predictions, responding to and interpreting data that they will collect through careful measurement.

Visualisation and generalisation skills will be used to find an optimum solution to the problem, before exploring how soap solution can provide an enlightening source of information.

The resources needed for this activity can be constructed from readily available materials, details of which are given.

Resources

- Whiteboards and dry-wipe pens
- Copies of worksheets from templates
- Bucket(s)
- Washing up liquid
- Perspex soap film frame (as illustrated) – these can be constructed using two rectangular pieces of Perspex with four spacers attached at the corners of a square. These can either be glued in place, or constructed from small suction cups (two can be glued together so as to attach to both surfaces), for greater versatility
- Access to a computer and data projector with a spreadsheet programme, so that pupils' results can be quickly graphed
- Protractors
- Glycerine (available from chemists) – an egg-cup full can be added to the soap solution to help the bubbles and soap films to last longer

Introduction

What sorts of considerations do road planners have when they are trying to build roads to connect towns or cities together? Gather pupils' thoughts on different aspects, such as cost and environmental impact and discuss what the priorities for the road builders may be.

Activity one

In what ways might we connect four 'cities' that sit at the corners of a square? It must be possible, using the roads they have designed, to get from any one city to any other. It is perfectly acceptable for the route to take them through another city. How can we achieve this using the shortest total road length? This is the minimum road length and we are looking to minimise the problem. Pupils should design suitable routes on the template sheet, then measure and record the total road length for each. Configurations such as a 'Z', 'N' and 'X' make a good starting point, and pupils should be encouraged to recognise suggestions that are equivalent to one another. When pupils have come up with a few designs, gather these together to discuss which are most efficient (in terms of having minimum length).

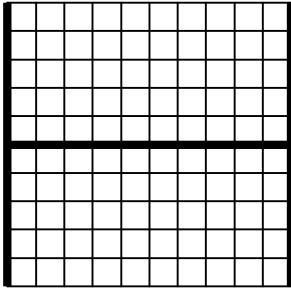
Mention any road layouts where a whole road could be removed and the layout would still be sufficient.

Analysing their designs in this way provides a good starting point for discussing practical aspects of the resulting network. If these points are real cities, what could the problem be if cars have to pass through several on the way to their destination? Additionally, ask pupils to consider whether the shortest total road length would mean the shortest journeys for drivers. It should quickly become apparent that they do not.

Review any general principles pupils have identified in preparation for refining their designs. Ideally, at this stage, pupils should be aware that an 'X' provides a fairly efficient design, though not the most efficient, as they will go on to discover in the second activity.

Activity two

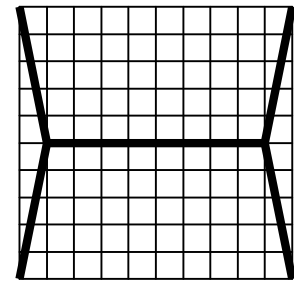
In activity one, pupils will probably have created designs in which each road section starts and ends on a city. In this activity they need to consider alternatives.



Show pupils an 'H' configuration of roads. Can they quickly work out the length of road that is required?

Now show the configuration below. How is it like the 'H'? What will happen to the total length of road as the length of the horizontal line is reduced? Encourage pupils to discuss the fact that the line will become shorter, but the others will not stay the same.

As a group, pupils will gather data on the effects of shortening this 'road' on the total road length of a network connecting four cities, set at the corners of a 10cm square. The template later on can be used for this. Each individual or pair of pupils can investigate the total road length when the length of the horizontal 'road' is reduced by a certain length at each side. The resultant lengths can be found using rulers to measure. Ensure that different pupils or groups are looking into different arrangements so as to build up a good bank of data.



As these results become available, they can be put into a spreadsheet and used to form a line graph showing how total road length depends on the length of the horizontal 'road' as it approaches zero (in which case, the total length is that of an 'X'). Explain how the line graph shows the minimum. Pupils should recognise that the minimum total length does not occur at that point.

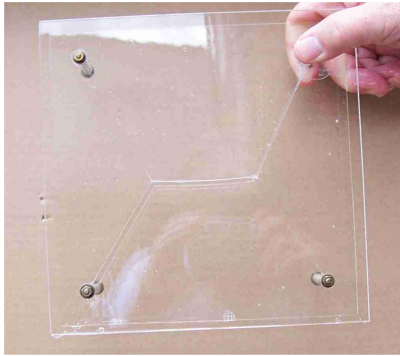
Activity three

What do pupils know about soap bubbles? What shape do they generally form? Encourage pupils to think about why they make similar shapes each time. Explain that the soap films actually make the smallest area that is needed to enclose the air in the bubble. Tell pupils that....

- The solution they will be using is a mixture of washing up liquid and water (and glycerine, if used)
- It is what's known as the surface tension of this solution that helps bubbles to form
- Because of the surface tension, the layer on the surface of the bubble acts in as if it's elasticated
- Compare this with blowing up a balloon – the balloon will form a round shape to allow fit the air in with the smallest surface area

- So the bubble solution will find the shortest way to connect the four ‘cities’ because it wants to reduce its surface area

Pupils should dip the soap film frame into the liquid and see the resulting shape



Using a protractor, pupils can measure the angle and will find that each of the angles equals 120° .

Measurement of the resulting path lengths can be used to see how well it corresponds to the information that they found in their diagram investigation earlier on.

Extension activities

There are many ways in which these activities can be extended. Pupils may wish to consider how roads could be planned to connect four cities sitting at the corners of other quadrilaterals, or to suggest what might provide the shortest road network for connecting three cities. This would allow them to find out whether the 120° angles hold for different layouts of cities. Pupils can use transparent sheets marked with three 120° angles around a point, as shown.

Overlaying these (two are needed to find the network for four cities) and moving them until a position is found in which roads can connect all of the cities.



Find out more

These activities have involved considering how soap films can illustrate minimisation in 2d. Studying soap films formed using 3d shapes can provide a wealth of exciting and thought-provoking activities. A Magic Mathworks Travelling Circus guide to investigating soap films can be downloaded from the Ri website, or obtained by emailing maths@ri.ac.uk. To find out more about the topic, Cyril Isenberg’s 1978 book ‘The Science of Soap Films and Soap Bubbles’ is recommended.

Activity one

Design some motorway networks that connect the cities in their groups of four. Measure the total length and record below the design.



Activity two

